3: DESIGNING A TECHNOLOGY BASED LEARNING ENVIRONMENT FOR PLACE VALUE USING ARTIFACT-CENTRIC ACTIVITY THEORY

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Modern interactive media provides technology that might be able to enhance teaching and learning of mathematics. Mathematics education must take into account the induced changes through this technological progress, as it cannot deny the existence of smartphones and tablet computers. Today, it is far more likely that a student has such a device on their person than it that he carries paper and pencil, a ruler or even a piece of string. While it is not at all clear that this is an improvement, it is a fact that must be taken into account when designing learning environments and manipulatives for teaching and learning mathematics.

A common fear is that students are going to be reduced to “answering machines” that are only required to tick the right answer in a multiple-choice test. This is supported by the growing number of assessments that reduce the mathematical competence of a student to a score in a standardized test. A climax of this development is the categorization of exercises into difficulty levels according to their solution rate in such a test: Is an exercise difficult just because many people cannot solve it correctly? This detachment of the actual content of an exercise from its analysis is hard to justify. Additionally, it counteracts and devalues good teaching – if students can solve exercises better, the exercises become easier by definition.

Activity Theory offers an alternative to test-orientation in psychology, and through this also a methodological alternative to quantitative empirical research. As stated in the preface to (Leont’ev, 1982):

> Die methodologische Unhaltbarkeit derartiger Tests ist offenkundig. […] Es ist unschwer zu erkennen, daß sich hinter einer derartigen Überführung einer methodischen Technik in eine selbständige Disziplin, wie sie mit der Testpsychologie entstanden ist, nichts anderes verbirgt als der Ersatz der theoretischen Untersuchung durch grobe Pragmatik.

In other words: The methodology of testing is just a coarse pragmatic replacement for theoretical analysis. In our work with multi-touch technology[2] we want to find out how to improve teaching and learning in mathematics through interaction with virtual manipulatives. As a preliminary step we have to understand how the interaction with such devices takes place (Ladel & Kortenkamp, 2013). With the help of the ACAT framework (see the Methodology section) based on Engeström’s (1987) work, we were able to focus on the activities of the students with the virtual manipulatives within a social setting. Here, we will discuss the design of a virtual place value chart and how the activity with such a virtual manipulative differs from that with traditional media.

[2] Multi-touch devices are able to report several touching fingers or objects on a screen, as opposed to single-touch or mouse-driven devices that are capable of reporting one (x,y)-coordinate on the screen and the information whether a button is pressed or not. Multi-touch devices are available in all sizes, ranging from smartphones via tablet computers to large-screen interactive whiteboards.

While empirical research about the effectiveness of digital learning environments or software for teaching and learning is definitely necessary, we reject research that accepts technology and in particular software and the application design as a given fact that is unchangeable. In contrast, we still see mathematics education as a design science (Wittmann, 1992) that not only analyses but creates learning environments.

PLACE VALUE AND NUMBER REPRESENTATIONS IN MATHEMATICS EDUCATION

The (decimal) place value system is a central pillar in basic arithmetic. Arbitrary large numbers can be represented uniquely by a finite set of digits. The decimal system allows for efficient ways to

- *Count*, as we can represent an infinite number of numbers with a finite set of words
- *Compare,* as the map from numbers to numerals is unambiguous and two numerals are represented by the same word, and there exists an algorithm to decide with of two different representation is larger by comparing two numbers starting at the highest place value
- *Add, subtract, multiply and divide*, using algorithms for written methods.

The underlying process of repeated bundling is a key factor for the uniqueness. We can bundle a cardinal representation of a number by bundling in tens, and tens of tens, and tens of tens of tens, … until no further bundling is possible. This process will always lead to the same number of (less than ten for each place) bundles. The bundling activity is also reversible, which is necessary not only for written subtraction, where we "borrow" from a higher place, but also for decoding the decimal representation into a cardinal conception.

Mathematically, the repeated bundling harnesses the power of exponential growth. The representation of a quantity n is possible with $\log_{10} n$ places using 10 digits. Here we clearly see the power of the decimal system that reduces large quantities into small representations. The product of two numbers is always representable through another number.

Another important aspect of the place value chart is the flexible interpretation of numbers that is induced by grouping digits differently. The number 3247 can be interpreted as 3 thousands, 2 hundreds, 4 tens and 7 ones (3|2|4|7), but also as 32 hundreds and 47 ones (32|47), 3 thousands and 247 ones (3|247), etc. For flexible arithmetic strategies it is helpful for children to be able to do calculations that are based on such decompositions, as opposed to using only written, algorithmic arithmetic that is place-based and only considers single digits.[3]

[3] Selter (1999) proposes to distinguish Zahlenrechnen (arithmetic with numbers) and Ziffernrechnen (arithmetic with digits). Usually, mental arithmetic is within the domain of Zahlenrechnen, and written algorithms fall into the Ziffernrechnen domain. Mental arithmetic supported by written notes that do not follow the standard algorithms for place-wise calculations (in German: halbschriftliches Rechnen, semi-written arithmetic) belongs to the domain of Zahlenrechnen and can benefit from flexible interpretations of place value charts. See (Benz, 2005) for details.

For written, algorithmic arithmetic it is necessary to understand the processes of carrying over and the reversal, borrowing from a higher place. This corresponds to replacing bundles of ten objects with a single representative or vice-versa. Our final goal is to support real “understanding rather than procedural proficiency”, supported by the instructional environment in the sense of Hiebert and Wearne (1992).

METHODOLOGY

We use the ACAT framework (Artefact-Centric Activity Theory, Fig. 1) to analyse the situation in which we are going to use a virtual manipulative for improving children’s understanding of place value.

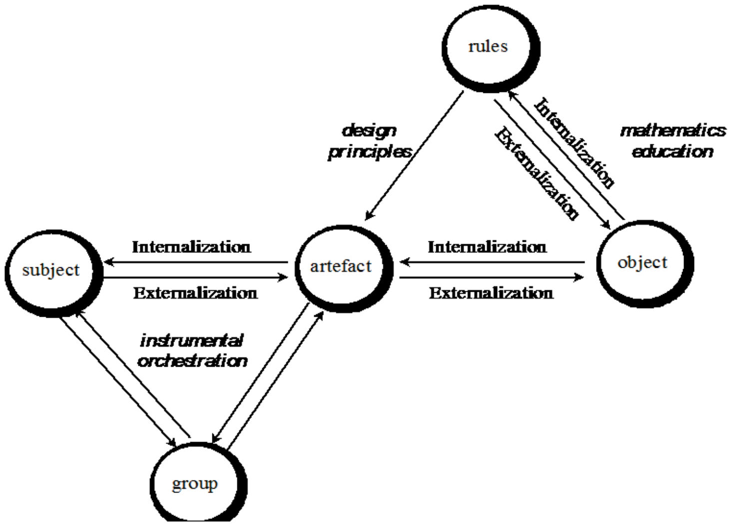

Figure 3: The ACAT Diagram

The actions carried out with that manipulative should support the mathematical design as described in the preceding section:

- Children should be able to place undistinguishable objects[4] in various places of the virtual place value chart.
- Moving an object from one place to the other should not change the value of the number represented, but initiates either a bundling or de-bundling.

Our goal is to help students to become fluent with these actions, such that they become operations for them. For a proper understanding of place value and for using flexible strategies when calculating, as well when doing written arithmetic, it is mandatory to do these operations then without the help of a supporting virtual manipulative.

[4] Gerster and Walter (1973) describe a 11-step abstraction model for the decimal system from grouping to representing numbers by sequence of digits. They emphasize the importance of the 9th abstraction which uses undistinguishable objects that differ in value only through their placement.

ANALYSIS

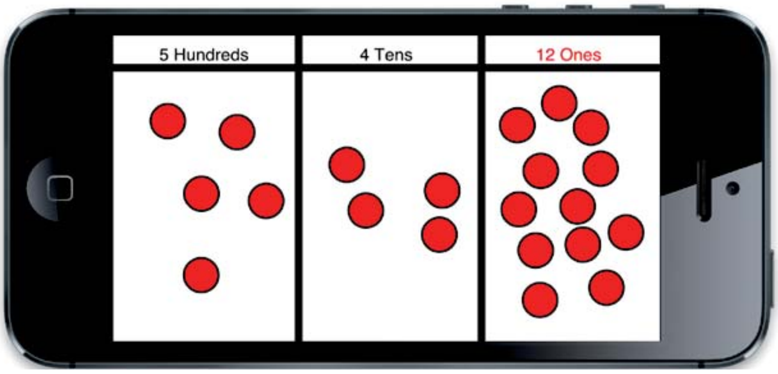

Figure 4: The virtual place value chart on the iPhone. Application available at https://itunes.apple.com/app/id568750442

Let us quickly review the main axis of ACAT (horizontal axis in figure 3): The subject – a student – manipulates an object, the number within the decimal system, by placing moving virtual tokens on the artefact. The artefact (the virtual place value chart) responds to these actions by showing the tokens and the number of tokens in each place. The properties of the decimal system are encoded into the artefact through its response when moving tokens from one place to the other: If a token is moved to a lower place, it "explodes" into the corresponding number of tokens, i.e. when a token is moved from the thousands' place to the tens' place, it will become 100 tokens. If a token is moved from a lower place to a higher place, the artefact will try to bundle the necessary number of other tokens and merge them into one, or it will refuse to move the token. Thus, the decimal system is encoded programmatically into the artefact.

A major difference between this design and traditional place value charts is the possibility to operate with the tokens *while keeping the represented number unchanged* [5] .This matches the mental operations necessary for addition and subtraction: When calculating 72 minus 47 a student should be able to use the strategy 12-7 and 60-40, which uses such a de-bundling of a ten-token into ten one-tokens. It is important to note that the virtual artefact is different to the traditional one here: When manipulating real tokens in a place value chart, children do change the represented number, which leads to questions like "when you move a ten-token to the one-place, how does the number change?" — Note that this activity helps children to understand the design of the traditional artefact, while our modern approach emphasizes the human activity and is ruled by the object (i.e. the numbers), not the artefact. The children are supported in their transition from "just moving tokens" to an operation that enables them do reach a higher level when operating within the number system.

[5] Our experience is that this difference is a key point of misunderstanding for most mathematics educator when evaluating our digital artefact. Only digital media allows for such a design, and many exercises built on the traditional place value chart no longer make sense with the new tool.

CONCLUSION AND OUTLOOK

In this brief overview we outlined how we could base the design of a digital artefact on a theory that answers some methodological drawbacks of pure quantitative methods. The view through activity theory is necessary to change the design from the traditional, non-digital design that emphasizes the tool and its response to human actions into a design that supports the cognitive processes of the children and respects the mathematical foundation of the tool.

As a next step we are currently designing and implementing a virtual manipulative that supports adding and subtracting in a subdivided place value chart, which gives rise to other possible actions that should become operations. In particular moving between the summands in addition should lead to discordant change (German: *gegensinniges Verändern)* and moving between the operands in subtraction should lead to concordant change (*gleichsinniges Verändern)*, as this will leave the sum (respectively difference) unchanged.