MATHEMATICAL SITUATIONS OF PLAY AND EXPLORATION AS AN EMPIRICAL RESEARCH INSTRUMENT

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Mathematical situations of play and exploration are being used as an empirical research instrument in the long-term study erStMal (early Steps in Mathematics Learning) in the IDEA Research Center at Frankfurt on Main[1]*. The communication has been designed as situations for encountering mathematics, which enable children to discover math in a playful context. Special description patterns in the form of Didactic Design Patterns have been developed for the comparison of the situations. For this reason the mathematical situations of play and exploration are developed as open learning opportunities and conversation opportunities on the one hand and as an empirical research instrument, on the other hand. Therefore these situations can arrange on the continuum between the poles of instruction and construction.*

INTRODUCTION

The mathematical situations of play and exploration in the erStMal project are used to create a framework for the mathematical discourse by groups of children (in pairs or small groups of four children) together with an adult. These mathematical situations, which are videotaped, form the basis for qualitative data analysis of the development of a theory of mathematical thinking. In contrast to the standardized testing procedures in the field of development diagnostics (Kienbaum & Schuhrke 2010) and the mathematical support diagnostics and learning assessment (Peter-Koop & Grüßing 2011; Steinweg & Gasteiger 2008), the mathematical situations of play and exploration give the children opportunities to express their mathematical (creative) potential and make their development over time visible. From special interest in this context are the connections and reciprocal relationships what are produced between the different mathematical domains by the children. At the current evolutionary state the mathematical situations have less normative character and conduce less to the learning assessment but provide data as a starting point for the development a theory of mathematical thinking within the framework of mathematics education (Krummheuer 2011; Vogel & Huth 2010).

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SITUATIONS FOR MATHEMATICAL PLAY AND DISCOVERY – SPACE FOR CONSTRUCTION

The mathematical situations of play and exploration (one adult with pairs or small groups of three to four children) focus on one mathematical task or problem, which is presented in a playful or exploratory context according to the age of the children. This format was chosen because the discussions oneself distinguish by a relatively high degree of close communication (Koch & Österreicher 2007, 350; Vogel & Huth 2009, 38/39). On the one hand the increased concentration of communication can be used motivationally and on the other hand for the discursive adaptation of the mathematical problem (cp. Vogel & Huth 2010, 184/185). The mathematical instruction is selected from one of the following five domains: numbers & operations, geometry & spatial thinking, measurements, patterns & algebraic thinking or data & probability. The instruction is then presented in a discursive teaching and learning environment by using appropriately selected material and linguistically and gestural stimuli. The results of empirical studies (Clements & Sarama 2007; Sarama & Clements 2008; Schuler & Wittmann 2009), concrete tips for the work in day care centers from books for the practise (e.g., Noenisch & Niggemeyer 2007; Benz 2010) as well as publications in which concepts are described for the creation of mathematical teaching and learning environment (Hülswitt 2006; Wollring 2006) and publications in the field of the early education (Fthenakis et al. 2009; Korff 2008) supply approaches and encouragement for the development and conceptualization of the mathematical situations of play and exploration in the erStMaL long-term study.[2]

In the mathematical situations of play and exploration a great importance is attached to the application of material, which encourages the process of playing and exploring. Thereby, it is important to consider the manner in which these material provokes, accompanies and supports mathematical activities and, as a consequence, mathematical thinking. The activity-oriented approach described by van Oers (2004, 316/317) for early mathematical education, which touches on the cultural-historical approach for learning and development of Lev Vygotsky, assumes that mathematical thinking is not limited by genetics. Mathematical thinking arises from the idea that "activities resulting from specific actions or situations" will be designated as "mathematical" by the adults (teachers or parents) (ibid., 317; translated by R.V.).

[2] It is only called a selection of the literature, which was evaluated in the preparation of mathematical situations of play and exploration. The development of the mathematical situations is continued. In this context it is also an ongoing process of feedback to the current discussion of mathematical education in child-care centers and the transition to primary school.

> "These activities, in turn, achieve a certain meaning in the understanding of the individual. The process of selection, designation and assessment of certain activities as 'mathematical' is in principle a socio-cultural process. […] Consequently, mathematics is learned through social interaction and that in the context of meaningful activities" (ibid., 317) (translated by R.V.).

Meaning is not attributed to the materials themselves for the relevant activities in this theoretical context, but rather given a meaningful value by the people participating in the situation (cp. ibid., 321).

> "Learning processes are required to be able to use an aid properly. The behavior of the students is supported by the aid in a manner specific to the culture in such a learning process, and simultaneously, the advantages that are connected with the culture structure of the aid become obvious" (ibid., 312; translated by R.V.)

This perception of mathematical thinking follows the understanding of mathematics as a culture on the one hand (van Oers 2011), while on the other hand, we assume that mathematics will be learned through social interaction. The approach of Krummheuer and Brandt (2001) of an interaction theory of mathematical learning, shows in which way the real mathematical issue between the involved persons interactively are negotiated. It is worked out, how the learning of mathematics constitutes itself in the process of interaction. The way of the participating in this process of discussion gives explanation about the fact which learning rooms arise for the involved learners.

The perception of mathematics as an independent cultural orientation system is not new and has been handled in a special manner by ethno-mathematics (cp. Prediger 2001, 126). For learning mathematics, this means that points of access into this culture of mathematics must be found with regards to the way of thinking, the values and the acting in this culture. Prediger (2001) introduced one such approach in her "Concept of Intercultural Learning".

> "Mathematics instruction should be used to teach students to know and to understand mathematics as an own culture. To do this, they should experience this culture and socialize as much as possible within it so that they can move safely within certain limited areas. The acquisition of implicit knowledge about the approaches, standards and values is also necessary" (ibid., 129; translated by R.V.)

Above all, intercultural overlap situations, in which the culture of mathematics meets other parts of cultural life, provide students the ability "…to gather their own experiences in those overlap areas between mathematics and the world" (ibid., 130; translated by R.V).

Learning mathematics in such intercultural overlap situations depends upon interactive processes, into which the participants bring and develop their interpretation of the situation and exchange them with others (cp. Vogel & Huth 2010). The participants are familiar with mathematical culture in variably comprehensiveness and with it in dealing with the materials, language and gestures and their interpretation as aids to mathematical thinking.

The purpose of the mathematical situations of play and exploration of erStMaL is to create action spaces that make encountering mathematics as a culture possible. The mathematical discourse encouraged as part of the situations of play and exploration should permit the children to participate in the mathematical concepts what are presented in a multi-modal manner by the guiding adult. Participation in the sense that a learning space will be created, where "knowledge learning" and "human being learning" and in the same way "acting learning" and "common life learning" can be connected (cp. Wulf & Zierfas 2007; translated by R.V.) and a narrative argumentation become possible (cp. Krummheuer 2011; Tomasello 2008; Bruner 1986). Such a discourse format is significant for an overlap area between mathematics and the children's world of experience (cp. Prediger 2001).

To implement this goal, the conceptual design of the mathematical situations of play and exploration has been selected such that the various learning environments have their origin in one of the mathematical domains what are relevant for the erStMaL project. This is expressed in a specific environment of materials and room, which is allowed by the conservatively chosen spoken language and gestures as well as encouragement of activity of the attended adult.

Above all the selected materials and aids constitute connection points for the kindergarten children, to join the mathematical interpretations of guiding adult or to make modified interpretations of the arrangement and to bring their representations as well as the storylines and rules of their child-like world into the situation. The materials and aids play the part of a cultural tool (Bodrova & Leong 2001, 9) in the spirit of Vygotsky (1978). They become a bridge between the strange culture (in this case, the culture of mathematics), in which experts (in this case the adult) demonstrate handling the material with reference to the task and deliver one interpretation with it, in which manner this material has significance for the problems, which are to be solved. Thereby, they create connecting factors for childlike thinking and their advancement and they introduce into thinking and activity processes of the relevant culture.

> "Higher mental functions exist for some time in a distributed or 'shared' form, when learners and their mentors use new cultural tools jointly in the context of solving some task" (Bodrova & Leong 2001, 9).

Like the mathematical problems, which are to be solved, the materials display various degrees of freedom. The degree of freedom should be described in a variety of manners for the various situations of play and exploration. Every situation shares a need for clearly described mathematical tasks that originates from the respectively selected mathematical domain. Already such shows various degrees of freedom that will be strengthened or weakened by the degree of freedom of the material. In the "animal polonaise" situation (see figure 1) for example, the mathematical problem consists of finding all of the possible sequences that can be made using three animals. The level of mathematical freedom in this is rather small, since the task does not allow for very much mathematical scope. The scopes are found rather in the processing of the mathematical task, which can be processed by means of various combinatory solution strategies. These, in turn, are connected with the available materials. In addition to the animals that can be placed in various sequences on a podium, cards with the animals will be made available at individual data collection points. They provide the ability to document the individual sequences discovered. In doing so, the question of how many such sequences are there can be approached systematically.

Figure 1: Animals and stage from the mathematical situation of play and exploration "animal polonaise"

If one considers the “wooden sticks” situation as an additional example of a mathematical situation of play and exploration then this mathematical task exhibits a greater degree of freedom. Patterns should be created, by placing the colored wooden sticks (see figure 2). Since the wooden sticks differ in color but not in shape, color is an obvious criterion for the development of the patterns and patterns can be created, by placing certain sequences of colors side by side (cp. Hülswitt 2006). Another possibility would be to create geometric shapes from the wooden sticks that, in turn, serve as a unit of pattern development. The intentions for the situations of play show that the mathematical task allows scopes, which are increased even more by the scopes in connection with the materials.

Figure 2: Material from the mathematical situation of play and exploration "wooden sticks"

Overall the guiding adult is urged to initiate the employment with the mathematical problem through open stimuli, but he should intervene so little steering as possible in order to provide the most possible freedom for the mathematical situations of play and exploration. Thereby to the children presents the possibility to choose a form of the adaption of the mathematical task, which is adapted to the situation. Thus they can in the process produce, connections to other mathematical domains or to other worlds of ideas.

SITUATIONS FOR MATHEMATICAL PLAY AND DISCOVERY – SPACE FOR INSTRUCTION

At the same time, these staging (the mathematical situations of play and exploration) should be repeated several times in the research context in such a manner that comparisons can be made. To be able to meet these requirements for openness and standardization, a uniform structure for the description of the situations has been selected in the form of Didactic Design Patterns (Wippermann & Vogel 2004), which will be designated as “design patterns of mathematical situations” in the future in the context of mathematical learning and thinking.

The situations will be described by the research team along a structure that has been specially developed for the mathematical situations of play and exploration for the erStMaL project as a recursive development process. The “design patterns of mathematical situation” are sub-divided into the following central groups of categories: (1) organizational aspects, (2) realization-related aspects and (3) mathematical aspects (see figure 1). The organizational aspects are important for the organization of the research. The realization-related aspects refer to the concrete implementation of the situations of play and the mathematical aspects support the guiding adult in their decision-making during the situation.

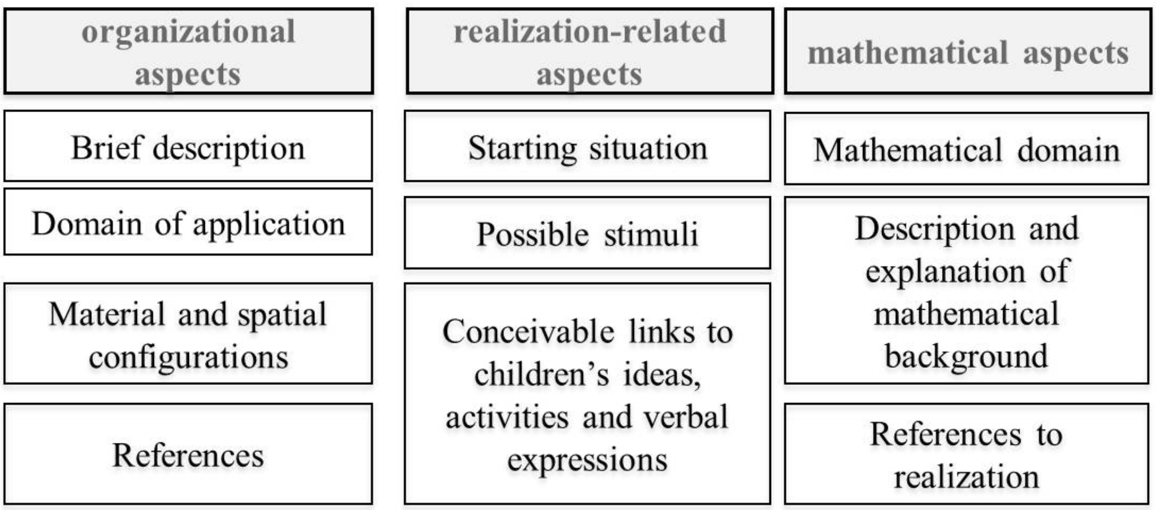

Figure 3: Structure for the “design pattern of mathematical situations” in the erStMaL-project

The individual descriptive categories are related to each other. Thus, a repertoire is described by possible stimuli in the realization-related categories. In the specific situation, selections can be made between them. Which of the possible stimuli for activities, gestures and spoken instructions in the “design patterns of mathematical situations” are appropriate must be decided in the situation and presumes a certain measure of mathematical knowledge in the respective mathematical domain and, furthermore, in the mathematical domains that might be used for creative solution of the task, as well as a good pedagogical feeling for accompanying the mathematical learning process.

The writing process required for the creating of “design patterns of mathematical situations” follows the form of a written template. This template will have questions about each category. These questions make compliance with the selected level of abstraction easier during the writing process. Writing the pattern itself follows a discursive process. This means that several people will review the “design patterns of mathematical situations” over the creation process and thereby lead to a form that can be used for data acquisition.

SITUATIONS FOR MATHEMATICAL PLAY AND DISCOVERY IN THE CONTEXT OF A LONG-TERM STUDY

The individual mathematical situations of play and exploration for the erStMaL long-term study will develop further from data collection point to data collection point in the context of the mathematical domain. This continuing development will occur from various points of view. Therefore, the goal of the research of development lines involves a certain requirement on consistency and variation for investigation. Concretely, this means that the situations of play and exploration must remain consistent within their mathematical frameworks, on the one hand, while taking the developmental changes, such as the cognitive and communicative potential of the children, into consideration on the other hand. In addition at each data collection point, the children acquire progressive experiences situations of play and exploration. These experiences can be included in the situation at the next data collection point. Thereby, appropriate continuing development must be designed specifically for each situation of play and exploration in consideration of its respective degree of freedom.

The continuing development of mathematical situations of play and exploration can add to the characteristic components of a situation of play and exploration described briefly here: of the mathematical task, the materials and the stimuli set by the guiding adult during the performance of the situation.

The advancement of the mathematical task can occur within the selected mathematical area by adding additional aspects. Thus for example, the children might be actively finding all of the potential arrangements of a certain number of animals in the “animal polonaise” situation at the first two points of survey (three animals at the first point, four at the second). The task will then be extended for the third data collection point by the requirement of documenting the sequences found with the help of cards, on which the animals are displayed. Optimization of the solution strategies for finding all of the sequences can result through such documentation. An extension at the fourth data collection point will then include limitations on the arrangement of the animals on the circus podium, such as always placing the elephant at the front. This will be introduced into the situations of play with the help of dice as random generator.

The “animal polonaise” example shows how that material will change parallel to the extension of the mathematical task. Thus, the first data collection point begins with three animals, which increases to four, then the addition of animal cards for documenting the various sequences and the addition of a random generator in the form of dice for creating conditions at the fourth data collection point.

Other situations, such as “wooden sticks” will remain consistent in their mathematical task and materials over multiple data collection points, since both the task and the materials contain a great degree of freedom for the solution. In contrast, the materials will first be changed for the fourth data collection point by extending the creation of patterns in the space. Flat wooden sticks will no longer be used, but rather wooden sticks of a certain height (rectangular prisms), in order to create spatial patterns.

The components of the multi-modal stimuli must continue to be developed in a certain manner by the guiding adult. Thus the initial stimuli continue to remain in the repertoire of the guiding adult throughout and will continue to be documented in the “design patterns of mathematical situations” for the subsequent data collection point. In addition, progressive stimuli will be supplemented from data collection point to data collection point. The guiding adult is confronted with often-difficult task of making concrete decisions about which of the stimuli formulated in the pattern should be used, or better be replaced by more appropriate, spontaneous stimuli.

For example, the guiding adult has the task of introducing the mathematical task in the mathematical “animal polonaise” situation using the non-mathematical context of the circus as the framework for the activity. This should encourage the children to discover the potential sequences of animals on a circus podium as a form of exploration and symbolic play. In this situation, the guiding adult has the task of deciding at each data collection point and for each individual group of children the extent to which the circus context will be used and in which intensity. By adding the animal cards at the third data collection point, explanations will be required on the part of the guiding adult, who will create a connection between the plastic animals on the podium and the animal cards.

CONCLUSION

The mathematical situations of play and exploration were developed as an empirical research instrument and are used currently in the research of mathematics education at the Goethe's university of Frankfurt/Main at the institute of mathematics education and informatics education (especially in the group of researcher in primary mathematics education). Also in the seminars in the mathematical primary teacher education, the teacher students work with the format of the mathematical situations of play and exploration. In these mathematical situations the teacher students can observe the mathematical work of children. In addition they can also analyse the video-taped situations by dint of detailed questions, which give an in-depth look on the mathematical learning of children. At the same time they can experience, what it means to accompany children in their mathematical learning.

The aim is to develop the mathematical situations of play and exploration in two directions: as a mathematical diagnostic instrument and as an adaptive mathematical learning space:

- as a diagnostic instrument
  The "design pattern of mathematical situations" can be used to describe the diagnosis situations and to categorise the situation observations and to stick in a mathematical portfolio.
- as an adaptive mathematical learning space
  In the "design pattern of mathematical situations" the mathematical educational potentials can be described.

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REFERENCES

Benz, C. (2010). Minis entdecken Mathematik. Braunschweig: Westermann.

Bodrova, E. & Leong, D.J. (2001). Tools of the Mind: A Case Study of Implementing the Vygotskian Approach in American Early Childhood and Primary Classrooms. Genf: International Bureau of Education. Online: http://www.ibe.unesco.org/publications/innodata/inno07.pdf [Stand: 15.03.2011].

Bruner, J. (1986). Actual minds, possible worlds. Cambridge MA: Harvard University Press.

Clements, D. H. & Sarama, J. (2007). Early Childhood Mathematics Learning. In F. K. Lester Jr. (Ed.), Second Handbook of Research on Mathematics Teaching and Learning (461-555). Charlotte, NC: Information Age Publishing.

Fthenakis, W.E., Schmitt, A., Daut, M., Eitel, A. & Wendell, A. (2009). Natur-Wissen schaffen. Band 2: Frühe mathematische Bildung. Troisdorf: Bildungsverlag EINS.

Hoenisch, N. & Niggemeyer, E. (2007). MATHE-KINGS. Junge Kinder fassen Mathematik an. 2. vollst. Überarbeitete Auflage. Weimar: verlag das netz.

Hülswitt, K. (2006). Mit Fantasie zur Mathematik – Freie Eigenproduktionen mit gleichem Material in großen Mengen. In M. Grüßing & A. Peter-Koop (Hrsg.), Die Entwicklung mathematischen Denkens in Kindergarten und Grundschule: Beobachten – Fördern – Dokumentieren (103-121). Offenburg: Mildenberger Verlag.

Kienbaum, J. & Schuhrke, B. (2010). Entwicklungspsychologie der Kindheit. Von der Geburt bis zum 12. Lebensjahr. Stuttgart: Kohlhammer.

Koch, P. & Oesterreicher, W. (2007). Schriftlichkeit und kommunikative Distanz. Zeitschrift für Germanistische Linguistik, 35 (3), 346-375.

Korff, N. (2008). Entwicklung, Diagnose und Frühförderung mathematischer Kompetenzen im Elementar- und Primarbereich. In D. Bönig (Hrsg.), Handreichungen zur Entwicklung der Mathematikdidaktik im Elementarbereich. Bremen: Universität Bremen.

Krummheuer, G. & Brandt, B. (2001). Paraphrase und Traduktion. Partizipationstheoretische Elemente einer Interaktionstheorie des Mathematiklernens in der Grundschule. Weinheim: Beltz.

Krummheuer, G. (2011). Die empirisch begründete Herleitung des Begriffs der „Interaktionalen Nische mathematischer Denkentwicklung“ (NMD). In B. Brandt, R. Vogel & G. Krummheuer (Hrsg.), Die Projekte erStMaL und MakreKi. Mathematikdidaktische Forschung am „Center for Individual Development and Adaptive Education“ (IDeA) (25-89). Münster: Waxmann.

Peter-Koop, A. & Grüßing, M. (2011) ElementarMathematischesBasisInterview für den Einsatz im Kindergarten. Offenburg: Mildenberger Verlag.

Prediger, S. (2001). Mathematiklernen als interkulturelles Lernen – Entwurf für einen didaktischen Ansatz. Journal für Mathematik-Didaktik, 22 (2), 123-144.

Sarama, J. & Clements, D. H. (2008). Mathematics in Early Childhood. In O. N. Saracho & B. Spodek (Eds), Contemporary Perspectives in Early Childhood Education (67-94). Charlotte, NC: Information Age Publishing.

Schuler, St. & Wittmann, G. (2009). Forschung zur frühen mathematischen Bildung – Bestandsaufnahme und Konsequenzen. In M. Neubrand (Hrsg.), Beiträge zum Mathematikunterricht 2009 (419-422). Münster: WTM-Verlag.

Steinweg, A.S. & Gasteiger, H. (2008). 3. Zwischenstandsbericht: Wissenschaftliche Begleitung der Implementierung der Lerndokumentation Mathematik im Rahmen des Projekts TransKiGs Berlin. Online: http://www.transkigs.de/fileadmin/user/-redakteur/Berlin/Steinweg/Bericht_WissBegleitungTransKiGs_Berlin_Dez08.pdf [01.10.2011]

Tomasello, M. (2008). Origins of human communication. Cambridge MA: MIT Press.

van Oers, B. (2001). Educational Forms of Initiation in Mathematical Culture. Educational Studies in Mathematics, Vol. 46, No. 1/3, p. 59-85.

van Oers, B. (2004). Mathematisches Denken bei Vorschulkindern. In W.E. Fthenakis & P. Oberhuemer (Hrsg.), Frühpädagogik international. Bildungsqualität im Blickpunkt (313-329). Wiesbaden: VS Verlag.

Vogel, R. & Huth, M. (2009). Können wir auch rot-rot-rot nehmen? Die Grundschulzeitschrift, 23 (222.223), 38-41.

Vogel, R. & Huth, M. (2010). "… und der Elefant in die Mitte" – Rekonstruktion mathematischer Konzepte von Kindern in Gesprächssituationen. In B. Brandt, M. Fetzer, M. Schütte (Hrsg.), Auf den Spuren Interpretativer Unterrichtsforschung in der Mathematikdidaktik (177-207). Münster: Waxmann.

Vygotsky, L.S. (1978). Mind in Society. The development of higher psychological processes. Cambridge: Harvard University Press.

Wollring, B. (2006). Kindermuster und Pläne dazu – Lernumgebung zur frühen geometrischen Förderung. In M. Grüßing & A. Peter-Koop (Hrsg.), Die Entwicklung mathematischen Denkens in Kindergarten und Grundschule: Beobachten – Fördern – Dokumentieren (80-102). Offenburg: Mildenberger Verlag.

Wulf, Chr. & Zirfas, J. (2007). Ausblick: Lernkulturen im Umbruch. Zur Bedeutung von Ritualen und performativen Praktiken in Lernsituationen. In In Ch. Wulf, B. Althans, G. Blaschke, N. Ferrin, M. Göhlich, B. Jörissen, R. Mattig, I. Nentwig-Gesemann, S. Schinkel, A. Tervooren, M. Wagner-Willi & J. Zirfas (Hrsg.), Lernkulturen im Umbruch. Rituelle Praktiken in Schule, Medien, Familie und Jugend (323-328). Wiesbaden: VS Verlag.

Wippermann, S. & Vogel, R. (2004). Communicating didactic knowledge in university education – "Didactic Design Patterns" as Best Practices. In L. Cantoni & C. McLoughlin (Eds.), Proceedings of World Conference on Educational Multimedia, Hypermedia and Telecommunications 2004 (pp. 3231-3234). Chesapeake, VA: AACE.