THE ELEMENTS OF MATHEMATICAL CREATIVITY AND THE FUNCTION OF THE ATTACHMENT STYLE IN EARLY CHILDHOOD

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The paper deals with the development of mathematical creativity of pre-k children who have social/emotional difficulties. Starting point is the longitudinal study MaKreKi (mathematical creativity with children) in which theories of mathematic education and psychoanalysis are amalgamated for the investigation of the development of mathematical creativity.

INTRODUCTION

In the research about mathematical creativity seldomly the early childhood is taken into count, although mathematical creativity already develops at preschool age (e.g. BECKER-TEXTOR 1998; KRENZ & RÖNNAU 1997). Therefore, central research question is under which social conditions does mathematical creativity in preschool children develop?

In general creativity begins with the instinctual attachment system, which unites mother and infant at the start of life. It differentiates adaptively, in response to anxiety, into the exceptional skills with materials, words and notes found among artists, writers and composers (BRINK 2000).

Up to now there is no clearly and generally valid definition for (mathematical) creativity. Present empiric works measure mathematical creativity rather in the mathematical product and neglect the creative process. Thus the central research question is, how does mathematical creativity express itself at the age of preschool and how is it observable?

THEORETICAL PERSPECTIVES

Mathematical Creativity

Mathematicians and researchers in mathematics education as well as psychologists have examined mathematical creativity under their various scientific viewpoints (HARDARMARD 1945; SRIRAMAN 2004). It became clear that a clarification of concepts of creativity is difficult and additionally complicated by its relationship to the concepts of intelligence, giftedness and problem solving.

With respect to the relative lack of current research the following analysis tentatively will deal with the following three aspects of mathematical creativity (SRIRAMAN 2004).

• *Choice*: Poincaré (1948) described as a fundamental aspect of mathematical creativity the ability to choose from the huge number of possible combinations of mathematical propositions a minimal collection that leads to the proof. Ervynck (1991) understands by mathematical creativity the ability to generate mathematical objects or the generation of a base idea for coping of a mathematical problem within a mathematical context. From this definition he derives the following characteristic features of mathematical creativity:

1. Relational: With the production of mathematical objects the individual has to discover conceptual links between two or several mathematical concepts, so that an interaction of ideas enters. The different mathematical ideas can be understood as single blocks, which can be combined differently.
2. Selective: With competition of different mathematical blocks the individual has to make a choice on one (at best for the most useful idea). This character is similar to Poincaré's choice metaphor.
3. Compressed/briefly presentably: The individual has to find the suitable words or symbols for the presentation of the mathematical ideas.

With regard to the age group of interest under this *choice* aspect of mathematical creativity the production of (unusual) relations between mathematical examination and experiences and the playful contact with mathematical methods is understood.

• *Non-algorithmic decision-making*: According to Ervynck (1991), mathematical creativity articulates itself not when routine and/or standard procedures are applied but when a unique and new way of solving a problem emerges. Ervynck refers to the creative achievement of mathematicians, who created something new for mathematics. With regard to the age group of 3 to 6 years old children there is still to clarify, what could be meant by a "unique and new" way of solving a problem. At first one is able to shift therefore the accentuation and speak of the "divergence from the canonical" (BRUNER 1990, p.19).

• *Adaptiveness*: Sternberg & Lubart (2000) characterize creativity as the ability to present an unexpected and original result that is also adaptive. *Adaptiveness* describes children's ability to accomplish unusual descriptions of a happening and to adapt the original core of meaning of this description to a new situation.

From a socio-constructivist point of view the individual ability of mathematical creativity develops in the course of many interactions with other members of the culture. Sriraman (2004) emphasizes that creative solutions do not come "*out of the blue*" (ibd. p.21) and

> *"the types of questions asked are determined to a large extent by the culture in which the mathematician lives and works. Simply put, it is impossible for an individual to acquire knowledge of the external world without social interaction"* (p. 21).

In the tradition of an interactional theory of mathematics education this cultural embedment is conceptualized as the situational phenomenon of the interactively accomplished process of negotiation of meaning and the situationally emerging taken-as-shared meaning (BAUERSFELD 1995, COBB & BAUERSFELD 1995, JUNGWIRTH & KRUMMHEUER 2006, KRUMMHEUER 2007).

Beside this situational, micro-sociological access the MaKreKi-team also refers to psychoanalytically-based attachment theory, in which a function of the culture is understood as an aspect of the relationship between mother and child. Every creative person produces a personal iconography, primarily as a result of attachment-induced anxieties, only secondarily in response to social ills and injustice. Thus creativity links individual life with the symbolic orders of meaning in society and culture, but it often starts with anxiety generated by a suboptimal attachment style at the opening of life. To typify and understand anxiety-inducing attachment style that necessitate, and answer to, creativity, is the task at hand. The attachment system is now widely studied in the life cycle, but little is said about creativity as a concomitant of this system (BRINK 2000).

Attachment theory

Attachment theory originates from Bowlby (1951) and postulates the central roll of attachment behavior for individual development. The theory proves that already the infant has attachment behavior. Even the infant attempts to find attachment and to use this later as a home base for its exploration of the world. Bowlby perceives the attachment system as the central source of motivation. The antagonism between attachment and exploration has a highly relevant explanatory power. Both systems cannot be simultaneously activated. If a child feels secure, it can activate his exploration system and explore his surroundings. If it perceives a danger, the attachment system is activated. The child interrupts its exploratory behavior and seeks safety by its parent.

Bowlby' s model has subsequently been further developed. The development of a test for the study of attachment behavior by Bowlby's colleague Mary Ainsworth was of great significance. In the so-called "strange situation", a standardized observation situation, the quality of the attachment of the child to its mother (or to its father) can be measured. Four attachment forms were described:

1. Insecure-avoidant: The "insecure-avoidant" child (A) experiences that its mother feels best when it shows no intense affects itself and behaves towards her in a controlled, distanced manner with a minimum of affect.
2. Secure: The securely attached child (B) has, thanks to its sensitive mother, a chance to build up a secure relationship to her in which the whole spectrum of human feelings in the sense of communication with another, that can be perceived, experienced and expressed.
3. Insecure-ambivalent: The ambivalently attached child (C) has spent its first year with a mother, who sometimes reacts appropriately, and is at other times rejecting and overprotective, i.e. on the whole, inconsistent and for this reason

she reacts in a way that is unpredictable for the child.

4. Insecure-disorganized attachment: The disorganized/disoriented attached child (D) could not build up a stable inner working model, as its mother (or father) suffered under the consequences of an acute trauma (for example, the dramatic loss of an important person). They were psychically so absorbed by this loss that they could hardly take up a coherent relationship with their infant.

Relating this approach to the topic of mathematical creativity of young children at risk, the results of empirical attachment research point to the fact that the shaping of realm-specific (mathematical) creativity can not only be localized in the social-economic framework or in the potentially stimulating mathematical contents in the child's milieu but also primarily in the type of attachment of the child to its parents.

Due to the above-mentioned antagonism between attachment and exploration behavior, it is plausible to assume that, above all, securely attached children will develop great joy in mathematical exploration and creativity. However, the experiences with child therapy show that some children, who have an inherent ability that can be used to express emotional insecurity or suffered trauma, that is apparent in so-called insecure-avoidant attachment forms ("A") or in insecure-disorganized forms ("D"), attempt to compensate these experiences through their special giftedness. These two competitive hypotheses will be further differentiated and more clearly studied in the research project MaKreKi.

METHODOLOGY

Short description of the sample and empirical approach

The sample of MaKreKi is based on the original samples of two projects that are in the larger study IDeA. One project is a study of the evaluation of two prevention programs with high-risk children in day-care centers (EVA; http://www.idea-frankfurt.eu/homepage/idea-projects/projekt-eva). It examines approximately 400 children. The second project is a study of early steps in mathematics learning with regard to immigrant children (erStMaL; http://www.idea-frankfurt.eu/homepage/idea-projects/projekt-erstmal). This project includes approximately 150 children. Thus the original sample contains 550 children often with a precarious childhood.

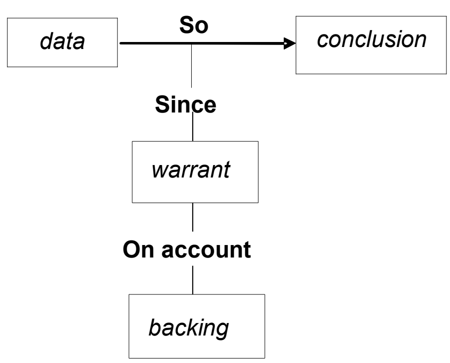
Due to the lack of tests to identifying mathematical creativity at preschool children, the MaKreKi-team developed a questionnaire in which the nursery teachers of the two original samples were asked, whether they knew children in their groups who show divergent and unusually sophisticated strategies while coping with mathematical tasks. In the combined sample of 550 children 40 children were identified who seem to creatively cope with mathematical problems.

Open mathematical situations of play and exploration (e.g. VOGEL/WIPPERMANN, 2004) were designed and applied in semi-annual surveys in pair and group settings in order to analyze the children's forms of mathematical creativity. These situations refer to the mathematical domains of number and operation, geometry, measurement, pattern and structures, and data analysis (SARAMA & CLEMENTS 2008). Every child particpates in two different situations of play and exploration per survey date and all mathematical situations of play and exploring are videotaped. These recordings are the basis for the intended interactional analyses.

Process of reconstructive analysis

Regarding the theoretical considerations and the attempt to identify mathematically creative moments in mathematical interactions of preschool children, in the following there is conducted an analysis of interaction, which refers to the interactional theory of learning (COBB & BAUERSFELD 1995, BRANDT/KRUMMHEUER, 2001). The method was devised by a working group round Bauersfeld in reference to ethnomethodological conversation analysis (GARFINKEL 1967). It focuses on the reconstruction of meaning and the structure of interactions (KRUMMHEUER 2011). Therefore it is proper to describe and analyse topics with regards to contents and the negotiation of meaning in the course of interactional processes. The negotiation of meaning takes place in interactions between the involved people. These processes will be analysed by means of ethnomethodology, in which is stated that the partners co-constitute the rationality of their action in the interaction in an everyday situation, while the partners try constantly to indicate the rationality of their actions and to produce a relevant consensus together. This is necessary for the origin of own conviction as well as for the production of conviction with the other participating persons.

The following analyses explore how children express and constitute their mathematically creative ideas. Argumentation processes can be reconstructed with the analysis of argumentation by TOULMIN (1969/1975). Four central categories of an argumentation are "data", “conclusion", “warrant" and "backing". Toulmin in 1969 has returned these functional argumentation categories graphically in a layout:



The general idea of an argumentation consists of tracing the statement to be proven back to undoubted statements (data). This relationship is expressed in the first line of the layout. Therefore this line can altogether be referred to as the inference of the argument. Such an inference requires a legitimation. Statements, which contribute to this, represent the warrant. Of another quality are those statements, which refer to the permissibility of the warrant. TOULMIN 1969 calls them "backings". They represent not doubtable basic convictions (e.g. the axioms in "mathematical" argumentations). Warrants and backings represent the depth of the argumentation. Arguments can be chained together in the way that an accepted conclusion can firm as data for a subsequent new argument.

Diagnosis of attachment style

For the diagnosis of the attachment pattern we apply the Manchester Child Attachment Story Task, so-called MCAST (GREEN et al. 2000). This is a story telling test that has good reliability and validity. A standardized dollhouse is used, the play of the child with the test coordinator is taped and later evaluated according to the test manual. In order to rule out the possibility that the behavior during the story telling, for example, is not determined by an exceptionally weak, cognitive ability, the MCAST is used in combination with an intelligence test. The Hannover Wechsler Intelligence Test in preschool age (HAWIWA-III), the German adaptation of the Wechsler Preschool and Primary Scale of Intelligence (2002) is implemented in the project. The test has been internationally shown to be reliable and valid. The test with its comparable subtests allows specific intellectual abilities to be observed over a longer time, for example during phases of therapeutic or pedagogic support.

The links between the type of attachment and mathematical creativity are examined by detailed interdisciplinary conducted case studies. These are longitudinal individual case studies in which the child and its development of its mathematical creativity will be observed over a period of 5 years. In this paper, first results of one individual case study will be presented.

FIRST INSIGHTS: CASE STUDY RENÉ

Psychoanalytic and attachment behavior analysis

René is a four years old boy who lives with his parents and his older sister in a small city. The teachers report that his father works fulltime in a computer firm and his mother part time.

Because of René’s very sophisticated language ability, the research assistant, who contacted René first, assumed that he was older than 4. In the Manchester Child Attachment Story Task (MCAST) René was very curious and highly motivated to cooperate, at the same time he demonstrated, however, in his facial mimic and body language a certain tension and restlessness. According to the 9 scales of the MCAST, René shows insecure-avoidant attachment behavior (A). In the HAWIWA René demonstrates average intellectual ability. Only in one performance subtest, "symbolic-figure" are his capabilities above average.

Analysis of René's mathematical solution process

The following summary covers the first results of René's mathematical solution processes in his first two episodes of the mathematical situations of play and exploration. The first situation is called "June Bugs" and it refers to the mathematical domain of numbers and operations. The second named "Dressing" refers to the mathematical domain of data analysis.

René's solution process in June bugs

In this situation the children can differentiate between similar objects, which differ according to their size and color. The objects are pictures of June bugs, which differ in size (small and large), in color (red, green, yellow), and in the spots on June bugs in two ways (circle, triangle, square and also by the sizes small and large as well). Beside René there are two persons involved: Lisa, a four years old girl from René's preschool and a member of our research team, who conducted the conversation with the two children.

This episode refers to the end phase of a collective processing of the task. René, Lisa and the member of the research team invented a familial system of description: The small June bugs represent kid-bugs and the big one mom-bugs, dad-bugs, or parents-bugs. During the period before this episode they also compared the number of cards according to their size and color and found out that all these subgroups are of equal number.

After this comparison the children realigned the cards around the round carpet, which is a kind of defined space for playing and exploring the cards.

In the center of this carpet the adult person several times puts a triplet of cards of the same color, but alternately of different sized cards and of different size and number of figures on top of the June bugs. Routinely the adult always opens a new problem with the question "which one doesn't belong" (WHEATLEY 2008).

The scene presented here is based on the following constellation:

René comes up with the solution that *both* June bugs with the many and small triangles do not belong. His justification has two aspects:

- Comparing the figures of the small and the big cards, he concludes, that the June bugs of the small cards should also only possess small figures on their tops.
- The two cards with the many and small triangles cannot exist in the system of the cards at all.

If one interprets these two warrants of his argumentation in his familial system of description one could rephrase it in this way:

- big June bugs have big figures because they are parents
- small June-bugs have small figures because they are children

- so, big June bugs with small figures do not exist.

If one understands the figures of the June bugs to be people's hands, René's argument is that parents do not have hands of the size of kids, this is impossible. They cannot be parents and children "at the same time", as he says.

With respect to the three aspects of mathematical creativity mentioned one can conclude: René's solution is based on a surprising *choice* of a familial system of description for the comparison of the June bugs. Hereby he does not create a somehow canonical combination of size and family-members:

	Big triangles	Small triangles
parents		
children		

He restricts this 2x2 table as follows:

	big triangles	small triangles
parents	ok	rejected
children	rejected	ok

René creates a *non-canonical* solution in which he combines the mathematical quantity size and the social and emotional quantity family.

Furthermore on the level of speech, he expresses this unusual *choice* by a linguistic adaption of the size of June bugs by use of a familial metaphor. He says that the big June bugs would be "already big". The wording of "big" can appear in the size-system of description and in a familial system of description. By combining "big" with "already" a process of change comes up: a June bug can grow to bigness and reach some features, which June bugs as a kid did not possess. This switch in his formulation is seen here linguistically as an *adaptive* achievement.

René's solution process in Dressing

In the following mathematical situation of play and exploration René and two other boys, Chris and Levent and a member of the MaKreKi research team are working on a combinatory situation of play and explorations, which begins with a short story about a paper doll called Kim. Kim is invited to a birthday party, but he does not know how to dress. He has got several articles of clothing: caps, sweatshirts and pants in different colors (blue, yellow and red). The children are invited to make different proposals of how Kim could dress up.

After finding some possibilities by the children the member of the research team suggests to put all Kims together that have caps of the same color. During this collection sequence René notes that all Kims are partners, but someone are looking different because they are only red, blue and yellow.

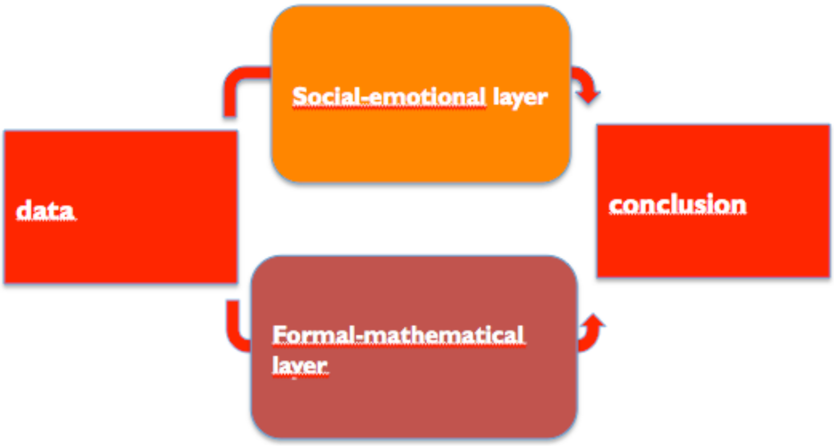
In this sequence René combines mainly two criteria:

1. Order Kims according to the color of their caps
2. Order Kims according to being dressed up in one color or in multi colors

Generally he combines both classification schemes with the non-combinatorial criterion of partnership, which is a surprising and unusual *choice*. We assign this criterion to the social-emotional level. René backs his argumentation concerning a social-emotional aspect (partnership between the dolls) and concerning a formal-combinatorial aspect (some of them are looking different, because they are dressed up in only one color).

SUMMARY

In both presented situations of play and exploration René's solution is based on a surprising *choice*, which includes the combination of formal-mathematical and social-emotional criterions. On the one hand we recognize elements of his specific mathematical creativity in the unusual combination of classification schemes.



Furthermore on the level of speech, he expresses this unusual *choice* by a linguistic *adaption* by using adquate metaphorization.

On the other hand we assume that the relationship of harmonious social emotional criterions are linked to his insecure-avoidant attachment type: he had to discover, that is his existentially important relationships he emotionally gets along the best, when he requests only few emotional (and cognitive) reactions from his parents and, instead of this, attempts to solve his momentary problems by himself. In this context he developed a form of mathematical creativity.

CONCLUSION AND PROSPECT

The analysis of the two episodes show, that the aspects of mathematical creativity mentioned in the theoretical perspectives (*choice*, *non-algorithmic decision making*, *adaptiveness*) are empirically useful for the characterization and reconstruction of mathematically creative solution processes of preschool children. To be able to perform a contribution for the empirical founded concept of mathematical creativity in the early childhood.

The link between the child's attachment pattern and it's mathematical creativity has to be explored further. Therefore analysis of parental conversations are planned to get a deeper knowledge about the relationship between the child and it's parents.

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