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Nice text by problem posing – two samples with many challenges

The author is going to present some two sample of problems that were used in real mathematical competition in Lithuania AD2011.

The sample itself of problem used in that competition was originally selected by the author and then rewritten and rephrased in Lithuanian using names which are more familiar or otherwise more attractive for a Lithuanian child.

Afterwards that sample involved was transfused into Russian. Strictly speaking it wasn't (and shouldn't be) a bare translation but an overwriting still applying the same frame of problems but already changing heroes involved and many other things connected with cultural background and linguistical realities.

So that transfusion proved itself to be connected with a lot of rather subtle things. The whole process took naturally some time and naturally encreased the real desire of the author to hear at least some opinion about the quality of the whole entertainment.

So the author presented that Russian, original text again, to some skilled persons and was only glad to get some rather positive estimations from several places – from Russia itself, but also from Ukraine and Israel.

In Ukraine that sample as the whole was translated into Ukrainian and is already published [1].

Alone that successful circumstance could already encourage the author not only to start with an English translation but also try with the time to have the sample in some other languages. Firstly, of course, I would like to have it in German, which is the language that I so eagerly studied many years ago.

The author would like to present that English transfusion to the participants of our Conference.

Probably the main thing, which accompanies the author in doing that, is, first of all, his deepest belief that some of these problems, if not all, appearing in that sample formally for higher grades, may be rather easily adopted or otherwise be suitable (or at least to be of some use) even for these youngest kids – e.g. by changing numbers, which are used in original problems.

On the same time the author is also deeply convinced that the “nice text” which he tries to employ in every problem (as well as in its solving) especially in that very young age is even more attractive and useful as in any other period of age.

During our conference the author intends to present also the German version of these two samples.

The author is trying to use nice texts by problem solving since many years and have published several booklets in English [c.f. 2-4].

The author would be very fond to hear any kind of opinion, critics and doubts connected with these things and similar matters.

13th Lithuanian Mathematics Olympiad for youngsters AD2011
concomitant with the 26th Lithuanian team-contest
Grades 5 and 6

**Welcomed by Maria Falk de Losada, the President of the World
Federation
of National Mathematics Competitions**

Vilnius University, Faculty of Mathematics and Informatics
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1. One morning the Hedgehog in Fog suddenly felt as if a cloud of sorrow had descended upon him. No one would have been able to predict what would happen next, but suddenly the Grizzly bear emerged before him with an immediate proposal to manage some simple (according to his clearly expressed opinion, anyway) problem. It ought to remain a secret that one of the modern devices that irritated the Grizzly bear deeply and which he carefully avoided was any kind of calculator. Even the humble Hedgehog, although in a much milder form, had often expressed similar attitude. At the same time the Grizzly Bear was full of compassion for all. In this case, he started the discussions whether this problem was indeed solvable by all, who declared at least a partial interest interest for it. In that case the Hedgehog in the Fog thought that it was indeed a problem for all, while the Grizzly expressed some doubts about all that.

But we've almost forgotten the problem itself. The problem was finding out (without using a calculator) how many integers are there in the first hundred of integers or between integers

1, 2, 3, ..., 98, 99, 100

when the sum of digits of the integer could be divisible neither by 2 nor by 5.

The hedgehog in the fog had immediately sunken down in the counting in order to find out the correct answer. It ought to be told, that he didn't have a habit of making mistakes in arithmetic, although at same time it could be added that he was a slow thinker.

So how many such integers in the first hundred are there with the sum of digits neither divisible by 2 nor 5?

2. After each solid dinner, when the Grizzly Bear felt satisfied (for the time being, anyway), he wasn't laying in vain – as some of us would expect – but always took a walk on another side of the river, where grasslands seemed to be endless.

On each walk he always happened to pass through a wonderful meadow, always having the form of a square and always being of different size. Each time when Grizzly laid his foot on that magic meadow, it immediately changed its size into some rectangular shape. One side of that rectangular proved itself to be on one third longer and another side, to the contrary – always 30 meters shorter.

Being very aware of those changes Grizzly Bear felt himself eagerly wishing to one day lay his feet on such a magic square meadow that the fence, which would be needed to surround it, would remain of the same length after that miraculous transformation of the parcel into the rectangular shape.

Is that possible?

If it is possible, then what is the length of the side of such a wonderful square size meadow and what is the length of that “invariant” fence?

3. The White Horse, although he was rather seldom seen in the company of other animals, was in fact highly regarded as a devoted partner and trusted friend of the Hedgehog in the fog. Also the Horse liked to appear as if descending from high above by always bringing strange problems of complicated origins. Solemnly speaking, from the first sight the Hedgehog in the fog usually was not that enthusiastic about solving them or even reading the formulation.

But in time he got used to getting involved in the process of solving, was always doing his best but if he wasn't able to achieve at least a slight progress, he would get very irritated and lose his temper with the outbursts of anger not always predictable.

Today the White Horse with the usual post also received a puzzle that looked quite difficult. It consisted from 16 pieces of joined letters and numbers. They looked exactly as it is indicated below or:

$a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4, d_1, d_2, d_3$ и d_4 .

In the fog the whistle of the Owl was so clearly distinguishable and at the same time a clear invitation for all who were of some scientific importance in that forest of clever animals, at least try to fulfill the following challenging scientific task: to put these jointed letters with the numbers into 16 entries of 4×4 square in such a way that in each row as well as in each column all these four letters

a, b, c, d,

as well as all these four integers

1, 2, 3, 4

were represented exactly once.

The Grizzly Bear, who appeared immediately, expressed clear doubts concerning the possibility to solve it. The Hedgehog in Fog on the contrary eagerly believed in the possibility to solve it, although without any scientific basis. He simply told that the puzzle was too nice not to be solvable. Only imagine: in every row, as well as in every column – all the digits and all letters without any repetition in any row or in any column.

Is that really possible? Or it is too good to be true?

4. An excellent mood of the Owl, who was always searching for answers to some puzzling question, or the outbursts of arithmetical fantasies of the White Horse usually were not so easy to predict. Still, when they took place, the Owl often started asking not just a single question but the entire series of them. Then even such an absent-minded dreamer like the Hedgehog in the Fog or the Grizzly Bear were completely aware that everything was well in the society of owls and even horses.

You might easily start solving an actual series of questions yourself. These were heard in practically every home in that clever forest. It sounded as follows (it also ought to be mentioned that the answers were always supposed to be presented to the Owl with the goal in mind).

(A) Is it possible to detect such an integer with the sum of digits not divisible by 6.

(A) Is it possible to detect some three consecutive natural integers with the

sum of digits in any of all three cases also not divisible by 6? The request for immediate answer is even more precise than it usually was.

(B) Is it ever possible to have a hope that there exist some six such consecutive natural integers with the sum of digits again not divisible by 6 in any of all six cases?

(C) To sum up - how to deal with the following extreme question of that resolute Owl: at most how many consecutive natural integers are there with sum of digits not dividing 6 in any of cases?

5.

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There is a token in the left most square in the lowest row in a 5 x 5 square. The Hedgehog in the Fog and the Grizzly Bear in turn are going to move it into the neighboring (sharing a common side) field. The Hedgehog will make his move first. The player who is not able to move the token into a field where that token hasn't been before, loses. Is it possible that any of these players may possess a foolproof strategy – that is that he always wins independently, despite his opponent's actions, and how should he proceed in such a case?

Grades 7 and 8

1. A white horse has from places unknown procured a dull and common, yet still 3-digital integer. If you ever believed that it would disclose its origin, then you should freely regard yourself as a deeply mistaken person. An owl, probably with some goal in mind but without any expressed cause, immediately permuted its last two digits and the grizzly bear, who actually was just passing by, added up these two integers in a blink of an eye.

The hedgehog in the fog then rather humbly asked what the sum was.

The owl, for reasons unknown, strictly declined to give a prompt answer. Instead it announced, that the result was already a 4-digit number and that he only agreed to announce its first three digits, these digits being exactly 173.

The hedgehog in the fog realized without delay that all that was actually worth doing was undertaking any possible and impossible efforts in order to deduce that number.

What number will he find if he remains as hardworking and consequent as always?

2. In an endless hollow of a very old oak the owl discovered some curious manuscript, containing a comical 100-digit number, written directly on the front page. It was clear that its first 49 digits were all 9's. Then the 50th digit followed, which unfortunately the owl was not able to decipher. The rest of the digits, except the last one, were all 0's. The 100th digit was again 9.

On the very last page of the manuscript the careful owl wasn't supposed to be astonished to find a message that this number was also a perfect square of another integer, which was claimed to be famous since the prehistoric times.

The hedgehog in the fog understood only too well that the Owl will not leave him be with his silly questions about the nature of that integer, so he immediately started with his attempts to:

(A) decipher that 50th unreadable digit of that original 100-digit number as well as

(B) find out the square of what integer might that original 100th integer be.

3. With remarkable admiration the hedgehog in the fog and the white horse once took a part in the competition of throwing darts to the target. The judge, the grizzly bear itself (with noticeable aplomb yet extremely carefully) provided all the necessary statistics. The owl, the one with the remarkable voice and very engaging, commented that entire happening also assisted him.

At first the Grizzly Bear clearly fixed that the Hedgehog in the Fog and the White Horse had made 5 throws each and the scores they had achieved were worth

10, 9, 9, 8, 8, 5, 4, 4, 3 and 2

points.

Then the Owl added (rather enthusiastically) that the general score of both after the first 3 throws was exactly the same. After a rather awkward pause it was revealed to the public that in its last 3 throws the White Horse achieved a score three time bigger that the Hedgehog in Fog.

Comparing those two contrasting announcements many of the spectators got remarkably excited and even claimed that such a big difference is impossible.

This uneasy atmosphere was diffused after a while, when the main judge – the raven - appeared. He listened carefully of tales about all these remarkably contrasting scores and after some hesitation silently announced that all the scores might be as they appear. “Moreover”, he added, “everyone, who wishes, is able to extract from the given data what marks have been achieved by both of them in their third throw”.

So what result did the Hedgehog in the Fog and the White Horse in their third throw achieve?

4. On one mysterious night the Owl started to press everyone who could hear him with the following, really challenging question - whether it is possible to find such a quadrilateral and also a triangle and to then get an octagon in their intersection.

Both the Grizzly Bear himself and the Hedgehog in Fog immediately felt fully prepared to prove that this was completely impossible. In other words, they wanted to prove quadrilateral intersecting with the triangle could never produce an octagon.

They were continuing their efforts till the darkness came on with their proof but they didn't found the slightest mark of it. They were already starting to discuss where these attempts may lead and suddenly....

What do you think happened?

Is it is possible to have an octagon as an intersection of some quadrilateral with a triangle, isn't it?

5.

T						

There is a token in the left most square in the lowest row in a 7 x 7 square. The Hedgehog in the Fog and the Grizzly Bear in turn are going to move it into the neighboring (sharing a common side) field. The Hedgehog

will make his move first. The player who is not able to move the token into a field where that token hasn't been before, loses. Is it possible that any of these players may possess a foolproof strategy – that is that he always wins independently, despite his opponent's actions, and how should he proceed in such a case?

6. The Hedgehog in the Fog and the Owl with a clear goal in mind spent full three days in heavy discussions about the possibility of an ice hockey tournament. In that tournament each team would be expected to play once with any of the other teams. Furthermore, in that tournament more than 5 teams should take part. But these were not all of the requirements. They also insisted that the team, who took the last place, should still win at least 25% of games and, moreover, the team that landed in the second place shouldn't win more than 40% of games.

In hockey, which for the Hedgehog in Fog appears to be a rather strange and even partly exotic game, each team gets 2 points for the win, 1 point for the draw and 0 points for the loss.

Is such a tournament possible or are all of the requirements together impossible to fulfill?

There is one thing that must be stated immediately. If you made a decision about the possibility of such a tournament, then the Hedgehog in the Fog and the Owl would not speak with you unless you prepared the table.

Is it possible to produce such a tournament table?

Literatur.

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