"I HAVE A LITTLE JOB FOR YOU"

RECONSTRUCTIONS OF FOLK PEDAGOGICAL IDEAS IN MATHEMATICAL INTERACTION PROCESSES WITH KINDERGARTEN TEACHERS AND YOUNG CHILDREN

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1 INTRODUCTION

With the publication of the international comparisons, early education has attracted the interest of scientists and the general public. In Germany, new plans for the education of children's day care centres are being devised, and most of the German states include mathematics as its own educational discipline. As part of the attention focused on this elementary field, a number of concepts for mathematical education are entering the market of day care centres, from the strict study methods of teaching-learning programs (primarily in the field of arithmetic) to the very open approaches of everyday routines and childish games that include potential for mathematical learning. But there are relatively few insights with regard to the current practices of mathematical education in kindergarten daily routines as a starting point for possible changes, both with regard to the implementation of concrete programs as well as to the practices generated in the kindergartens themselves for didactical materials, in the form of self-staged learning arrangements and free play.

In this paper, I will focus on mathematical settings designed by kindergarten teachers, which are part of the research project erStMaL (early Steps in Mathematical Learning[1]; see Brandt, Vogel & Krummheuer 2011). For the reconstruction of the situational dimension of the pedagogical aspects of the activities and the implications for the mathematical learning, I use the concept of folk pedagogy (Bruner & Olson 1996) and in supplement the instruction models described by Rogoff (1994) and Rogoff, Matusov & White (1996) (cf. Brandt & Tiedemann 2011 and Tiedemann & Brandt 2010).

2 THE RESEARCH PROJECT ERSTMAL

My empirical considerations are embedded in the research project erStMaL (Brandt et al. 2011). The project erStMaL is a longterm qualitative video study that researches

[1] The erStMal project has existed since 2008 at the IDMI (Institute for Didactics of Mathematics and IT) at the Goethe-Universität Frankfurt. It has been integrated into the IDeA (Individual Development and Adaptive Education of Children at Risk) Research Center, which was established at the initiative of the Hessian program for excellence LOEWE (Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz). The center is a collaborative effort between the DIPF (Deutsches Institut für Internationale Pädagogische Forschung), the SFI (Sigmund Freud Institute) and the Goethe University in Frankfurt.

the development of children's mathematical thinking under simultaneous consideration of the relationships between various mathematical areas and various social settings. The main research goal is to develop elements of a theory for understanding mathematical thinking processes of children between the ages of three and nine[2], regarding social-interactional and individual aspects of development in mathematics learning.

2.1 Theoretical Background

The research project is theoretical and methodological based on the Social Interactionism and the idea of negotiation of meaning (Blumer 1954, 1969) as well as on the ethnomethodological concept of local production (Garfinkel 1967) (for adaption of these concepts to mathematic education see e.g. Coob & Bauersfeld 1995; Krummheuer 2011a; Krummheuer and Brandt 2001; Brandt & Tatsis 2009).

The request involves tracing developmental lines in different mathematical domains. According to the research literature and the NCTM Standards (c.f. Sarama & Clements 2008) the domains of erStMal are *Numbers and Operations, Geometry*, *Pattern and Structures, Measurement*, and *Data Analysis (including theory of combination)*. Sarama and Clements (2008) postulate that it is important to focus on relevant domains and their relationships to support the development of mathematical thinking in early childhood. It is needed to get to know more about learning trajectories and to understand how kids understand mathematics, how they create mathematical ideas and how they express them. Thus, in the research project erStMal, we focus our research interest on following overlapping aspects of learning mathematics in early years:

- Interactional support systems for the acquisition of mathematical concepts and operations/processes and the participation in mathematical discourses.[3]
- Pedagogical and psychological aspects of the activities in the interaction processes.
- Development of mathematical concepts within and across different mathematical domains (conceptual change).
- Relationship of language and mathematics and multi-modal aspects of children's mathematical concepts in discourses within preschool and primary math classes (e.g., speech, gestures, actions, inscriptions).

As an orientation for our theoretical considerations of the development of mathematical learning, we adopt the concept of "development niche" (Super &

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[2] In Germany, this is the age-group of (mainly) starting with Kindergarten until the third year of primary school.

3 Following the idea of *Language Acquisition Support System* (LASS, Bruner 1983, 1986) we describe these interactional support systems as *Mathematics Acquisition Support System* (MASS; see Tiedemann 2010) respectively *Mathematics Learning Support System* (MLSS; see Krummheuer 2011b).

Harkness 1986; Harkness et al. 2007; cf. Krummheuer 2011b). With their development theory, Super and Harkness described the rehabilitation or developmental process of disable children in regard to the environment (cf. the ecological model of child development, Bronfenbrenner 1979). The child and its "particular set of inherited dispositions" (Harkness et al. 2007, p. 34S) are in the centre of the development niche, encompass by a system of cultural constructed environmental circumstances (physical and social settings, customs of child care and rearing, and caretaker psychology, ibid.; see figure 1), which influence the child's development. Due to aspects of the larger culture, children will be involved in different settings, different customs will be liven up by the participants of the social interactions and the adults will have different ideas of how children in general or a specific child 'is like'.

In adoption of this theoretical model, we consider allocational and situational aspects of these sub-systems of the development niche (see Krummheuer 2011b for more details of modification). Reconstructing the situational dimension of the pedagogical aspects of the kindergarten teacher activities, my focus is on situational aspect of caretaker's psychology as well as on the situational aspect of the customs of care, which we subsume to the educational dimension of the interactional niche of development. For reconstructing situational aspects, I use the concept of folk pedagogy (Olson & Bruner 1996) and the instruction models (Rogoff 1994; Rogoff et al. 1996), which will be described in the next paragraphs.

2.2 Folk pedagogy as situational aspects of the educational dimension in interaction processes between adults and children

The folk pedagogy introduced by Olson and Bruner (1996) provides a connecting factor for the analyses as one aspect of a cultural psychology (Bruner, 1997), which 'explains' which approaches should be possible and expected in educational situations as part of the daily routine.

> "… we are steered in the activity of helping children learn about the world by a body of assumptions that make up what we may call 'folk pedagogy'. (...) Watch any mother, any teacher, even any baby-sitter with a child and you will be struck at how much of what they do is guided by notions of what children's minds are like and how one may help them learn, even though they may not be able to verbalize their pedagogical principles." (Olson & Bruner 1996, p. 10)

Olson and Bruner describe four everyday educational concepts that are distinguished by the convictions about how teaching and learning work, which underlie the approaches and the manner in which the knowledge should be taught and learned. The point of reference for the designation of the concept is the child as the subject of learning, towards whom the activity of the adult model has been oriented:

- children as doers: "the acquisition of 'know-how'" (ibid. p. 16),
- children as knowers: "the acquisition of propositional knowledge" (ibid. p. 17),

– children as thinkers: “the development of intersubjective interchange” (ibid. p. 18) and
– children as knowledgeable: “the management of ‘objective’ knowledge” (ibid. p. 21).

While the *children as doers* concept is more oriented towards a craftsman-like learning process and seldom found in our scenarios, the other concepts can be reconstructed as potential bases for the approaches taken by the participants. Thus, these concepts will be described in more details. The following table shows the roles of the children and the adult model, and describes the expected room for learning.

	Adult	Child
children as knower	Knowledge Agent Expert (transmission) - presents facts, standards and learning rules - checks, evaluates and motivates - is responsible for reaching the goal	Knowledge Recipient Tabula rasa with the ability to learn - learns and remembers knowledge (application) - adjusts to the situation - fulfills the problem
children as thinkers	Discussion Partner Friend - designs challenging environments - tries to understand the child’s thinking and work with it	Knowledge Constructor Self-teacher - develops and integrates analyses into their own canon of convictions
children as knowledgeable	Information Manager Expert - enables participation in the culture	Knowledge Assimilator Knowledge Acquirer - oriented towards participation in the culture
	Mutual learning regarding mastery of a cultural practice	

The introductions of the kindergarten teacher to settings[4] that were typical for the subsequent overall situation serve to illustrate these three concepts. Generally however, transitioning between the different concepts within a whole situation is possible.[5]

knowers	**thinkers**	**knowledgeable**
		

[4] The three episodes will be analysed in more detail in paragraph 4.
[5] Although we frequently observe a stability of concepts within the situations (cf. Brandt & Tiedemann 2011), switching between the concepts can also be observed in some situations.

Don't start yet. I have a little job that I prepared for you.	Come have a look and see if you have an idea about what you can do with this.	Do you know what we will do today? (…) Let's make a butterfly together.

2.3 Instruction models and their relation to folk pedagogy

The three concepts of folk pedagogy of Olson and Bruner can be compared to the instruction models, which were described by Rogoff (1994) and Rogoff et al. (1996), observing parents in school settings. They describe three different instruction models:

- Transmission,
- Acquisition, and
- Community-of-learners.

For the concept *child as knowers* and the instruction model *transmission*, an adult or an expert is compulsory necessary for learning processes; the participation of the adult effects the learning process, whereas the child is more or less seen as passive and not responsible for its own learning. As explicit described by Rogoff et al. (1996), this delimited the possibilities of the children in their participating in the current situation as well in the participating in prospective interactions.

> "Students learn how to solve problems but not how to set them. They can produce correct answers but do not have experience examining how to determine what is correct." (Rogoff, Matusov & White 1996, S. 393)

The concept *child as thinkers* and the instruction model *acquisition* can be described as different forms of constructivism. The concept of Olson and Bruner is oriented to socio-constructivism with more emphasis to the exchange with other for the construction process:

> "Their understanding is fostered through discussion and collaboration, with each child encouraged to present her own way of constructing the subject at hand to achieve some meeting of minds with peers and teachers." (Olson & Bruner 1996, p. 18)

On the contrary, the dedicated instruction model *acquisition* emphasis the individual part of construction as a kind of autodidactic process and reminds more to radical positions of constructivism (e.g. von Glasersfeld 1996).

The last two conceptions (*child as knowledgeable* and *community-of-learners)* correlated in their orientation to cultural aspects of learning. The learning process is seen as a re-construction of cultural approved knowledge, which is in principal modifiable; thus, the (re-)constructions of the child are cultural delimited but not definite by objective knowledge.

The interrelations between the different concepts of folk pedagogy and the instruction model are summarised in the following overview (see Brandt & Tiedemann 2011 for more details):

Folk pedagogy (Olson & Bruner)	**Instruction model** (Rogoff et al.)	**Main idea of learning and teaching**
children as doers	-	Learning as acquisition of "know how" – demonstration and imitation of activities
children as knowers	transmission	Learning as acquisition of propositional and objective knowledge by transmission of facts and rules
children as thinkers	acquisition	Learning as individual construction of knowledge in exchange with others
children as knowledgeable	community-of-learners	Learning by participating in cultural practises with support from experts (in interaction with more knowledgeable)

Although the terms of Olson and Bruner seam not so neat as the terms of Rogoff et al., I prefer the terms of Olson and Bruner by theoretical reasons, whereas the main reason is the position of the child and its role in the centre of the concepts. This conceptualisation fits to the idea of developmental niche of Super and Harkness (see above), which also have the child in the centre. Thereby, folk pedagogy is embedded in the customs of care as well as part of the caretaker psychology.

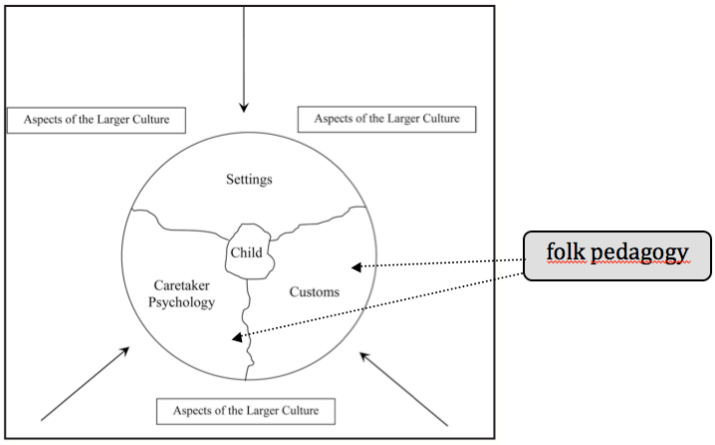


Figure 1: folk pedagogy and development niche[6]

3 DATA COLLECTION OF ERSTMAL

In the first survey period (2009–2011), we observed nearly 120 children longitudinal from the age 3.6 up to the school start in pairs or in small groups. The first survey period consists of four observation phases, conducted with intervals of around six months. The observations were carried out in 12 kindergartens in and surrounding of Frankfurt am Main. The socioeconomic status, gender roles and the language

[6] The figure of the development niche is taken from Harkness et al. 2007.

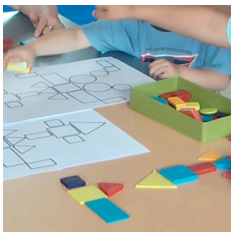
background of these children lead to an adequate sample that reflects the society of Frankfurt.

We observed the children in different social settings, in pairs (same age) or in small groups (mixed age), always accompanied by an adult.[7] The main observation settings were special settings of play and explorations with mathematical potential in different mathematical domains, which were designed for research purposes and were accompanied by a member of the research team. Furthermore, we observe settings, which were designed by kindergarten teachers as settings with mathematical learning opportunity, and which the kindergarten teacher themselves carried out with selected children. In this paper, I focus on the later social settings. Thus, the collection of these settings will be described in more detail in the following paragraph.

3.1 Data basis of this paper: the kindergarten teacher settings

In each of the four observation phases within the first survey period, ideally one of the kindergarten teacher in each kindergarten were observed with a pair of children and a small group of children.[8] The settings were designed by the kindergarten teachers and have been video recorded by a member of the research team.

We asked the kindergarten teacher to design play settings or learning settings with potential for learning in a specific mathematical domain, concerning our general research design: a) numbers and operations, b) geometry and sizes, c) pattern and structures, d) measurements and e) data analysis (including theory of combination). The kindergarten teachers were free in their design decision, only training-programs were excluded. The kindergarten teacher used existing commercial and self-created didactic materials, rule-based and learning games, puzzles, crafts, and construction materials for designing the setting; an overview of the variety is given by following table:

Numbers and operations	**Geometry**	**Pattern and structures**	**Measurement**	**Data analysis**
				

Several situations focus on a single area, while others cover the spectrum. E.g. the children used the glassy stones of the situation shown in the column *Data analysis* also for

[7] Typically, every child took part on two settings in each observation phase.

[8] Few of the planned settings were cancelled by different reasons (e.g. illness or drop out of a child or a short term refusal of the kindergarten teacher).

geometrical shapes (lines, circles and rectangles). Apart of this switching between the mathematical domains, counting processes are involved in the several situations (e.g. counting the glassy stones after categorising them in different groups or counting the 'small circles' which were needed for the construction in the situation shown in the column *Geometry*).

In addition to this variety of content, the situations in the scenarios determined by the teachers with potential for enabling learning mathematics also exhibit a broad time period. The shortest is a task of sorting animal figurines (data analysis: theory of combination) taking roughly five minutes. In comparison, a setting based on commercial material with basic, combinable geometric shapes for two and three dimensions puzzle and construction activities (geometry) led to a situation, which took almost 45 minutes.

4 ANALYSIS OF THE EMPIRCAL DATA

Designed by the kindergarten teacher as 'setting with specific mathematical learning opportunities', these different situations will be used for working out daily kindergarten routines of planed mathematical activities. Focussing the educational concepts and the embedded mathematical ideas, insights can be gained into the current mathematical teaching processes – omitting very open approaches of everyday routines in unplanned mathematical activities using favourable occasions.[9] Thereby, the situational production of the learning opportunities by the participants will be carried out with micro-analytical methods, based on a turn-by-turn analysis of transcribed sequences (see e.g. Krummheuer 2007, Brandt & Tatsis 2009).

For the focus on the emerging educational concepts, the idea of folk pedagogy as outlined by Olson and Bruner (1996) will be used as a "sensitizing concept" (Blumer 1954). Blumer differentiated between "sensitizing" and "definitive" concepts; thereby, "sensitizing concepts merely suggest certain directions along which to look." (ibid. p. 7) Using "sensitizing concepts" for the analysis of empirical data, these concepts describe a framework for the interpretation process:

> "Empirically related research questions ask less whether these concepts come to view in 'reality', but rather, how they orient one's perspective in order to interpret this 'reality'". (Krummheuer 2011, p. 82)

Looking to the local production of pedagogical and psychological aspects of the observed situations, I will use folk pedagogy (see above) as a framework for the micro-social analyses.

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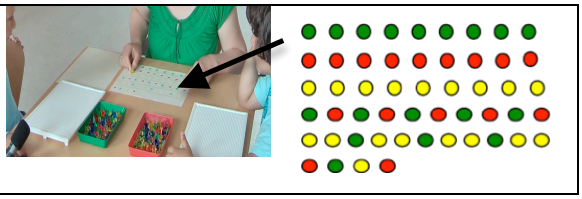
[9] In German kindergarten, you can find both forms in different conceptions of "kindergarten". Especially, in the last year before entering school, more planed mathematical activities will be offered to the child. Looking into the videos, there is an impression of 'routine' with such more formal settings by the participants (children and kindergarten teachers) in the most of the institutions of our sample.

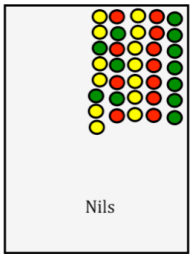
Considering methodological aspect of comparing analysis, the allocational aspect of content will be fixed for the selected situations; all selected settings are allocated to the mathematical domain *pattern and structures*. The social setting is variable: one of the selected settings is a group setting, the two others are pair settings. Behalf to the educational dimension of the interactional niche, the situations are allocated by the form of planned activities, and the situational aspect of this dimension is the focus of analysis. The situations are named by the materials, which the kindergarten teachers select for the setting.

4.1 Coloured Nails

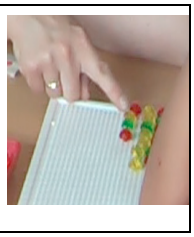
In this situation, the kindergarten teacher selects coloured nails (red, yellow and green), which normally were used for creating free patterns or coping pictures with concrete objects (flower, house etc.) on a pinboard. The kindergarten teacher prepared a paper with lines of dots (red, yellow and green) and a box with all materials. After entering the room with the two boys, she starts to arrange the prepared material from the box at the table as shown on the first picture of this scene.

F + N: *(both are grasping in the individual nail box)* TE[10]: don't start yet . I have a little job that I prepared for you *(showing the sheet of paper with the coloured dots)*	

Afterwards, she introduces the children in the task to copy her lines of coloured dots with the nails. Then, she guides the children line by line, covering the other lines and deciding, when the next line will occur.[11] She recognises, that Nils (N; boy on the right side) cope the forth line from the right side, but explicit allows this procedure "this is the other direction, but it's all right". Thus, she sets and controls the rules for the conducting of the task.



Nils copies the four coloured dots of the last line with nails, again starting on the right side and leans back after the forth nails: teacher: ●●●● Nils: ●●●● He seems satisfied with his conduction, but the kindergarten teacher intervenes:

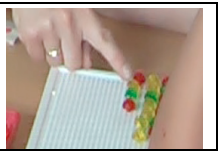
TE: so, here is free *(showing on her drawing)* but you have to fill it . now you have red yellow green . and now you have to start again here *(showing the red plugs one after the other) red and what is coming next (showing the yellow plug and than the „gap" at the end of his line)* N: well, there you need green TE: no, what is the next . what is next to red . look at your line . . what did	

[10] TE stands for kindergarten teacher.

[11] This is either the case of using the same number of nails on the pinboard as dots on the sheet of paper nor the case of building a line from the one side to the other on the pinboard as on her drawing. The decision seems situational and more or less accidental, perhaps oriented to the time.

you put after red here	

Whereas the children were obligated to copy the previous lines, the idea of the last line is the continuation of a pattern. Thereby, the teacher has a definite idea how to continue 'her pattern': starting again at the beginning of the line, whereas her last (red) dot belongs to the first iteration of the pattern (red, green, yellow) – changing the direction (red, yellow, green) seems again not important for her. Nils fits in to the idea of proceeding, but taking a green nail he adduces an own idea for continuation: a possible assumption is 'going backwards' (which means to produce a symmetric colour sequences).[12] Although the teacher confirms her interpretation, a few minutes Nils suggests to take a red nail at the fifth position (which could mean to iterate the whole colour sequences of the kindergarten teacher). But the teacher confirms again her idea and requests him to take a yellow nail in an interaction pattern, which reminds to the "funnel pattern" (Bauersfeld 1980; cf. Brandt 1997).

TE: well, *(showing the yellow plug)* what's the name of this colour N: yellow TE: then take it	

In this second part, again the kindergarten teachers sets and controls the rules of the procedure. Furthermore, she 'transmits' her idea of continuation as a kind of objective knowledge, although from a mathematical view, this is not the fact. Thus, she rejects the divergent ideas of Nils. These divergent ideas will not become topic of the interaction, but substituted by the 'objective knowledge' enforced by the adult as knowledge agent.

Generally, this situation could be assigned as a prototype of children as knowers.

4.2 Lot of things

In this situation, the kindergarten teacher selects an accumulation of different things in various numbers (diverse glassy stones, dices, counter, wooden sticks, etc.) and a special kind of placemats. She prepared all of these materials in the middle of the room (see 2.2, example 'children as thinkers'). Entering the room, she asks the children to choose a placemat and opens up the 'room' for the children's ideas: Come have a look and see if you have an idea about what you can do with this. Then, she sits down on the floor a little bit apart. In fact, all children start to arrange different things on their own placemat. The following table gives an overview of the beginning phase (Principle, the kindergarten teachers comments only very few of the children's activities; I pick up mostly all of her comments within this short period, whereas I skip some commentaries of the children.)

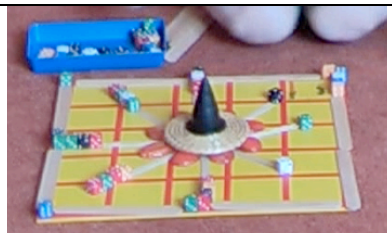
[12] Whereat it is not sure that his has any idea of a pattern by choosing the green nail.

time: 1:35 U: *(puts a big, red glassy stone in the middle of her round placemat)* look Nadine *(name of the kindergarten teacher)* TE: fantastic you have found a centre, like a Mandala, yes time: 2:06 B: (*puts a big, green glassy stone in the centre of her round placemat)* I have the big one time: 2:24 B: *(singing)* and I make a centre, too (*puts a big, green glassy stone in the middle of her round placemat)* TE: You have a centre, too . where is your centre B: *(showing the big green glassy stone)* there TE: that is the centre of the circle time: 2:40 K: where are the big one *(stands up, retrieves a big green glassy stone)* S: *(at the same time: puts a big red glassy stone in the centre of her quadratic placemat, without any comment)* K: I have a centre too. TE: and you have also a centre . . . a tetragon has also a centre . . and Sara has found her centre too	   

Within one minute, all children grasped the idea of "centre", which Ursula (U) brings into the discourse. This was one of the few utterances, the kindergarten teacher reacts directly and extensive, thus, she emphasis the mathematical idea of the centre of a geometrical figure. But she ties her comments to the activities and utterances of the children. Afterwards, the children create very different arrangement integrating 'the centre'. The children named their arrangements "clown", or "like a face" (if they not work without any comment), but nobody grasps the idea of "Mandala", which the kindergarten teacher linked to "the centre" of the figure.

A few minutes later (time: 6:20), she starts an own 'mandala-like' pattern on a quadratic placemat, but without any comment (see below). After a while, Karl (K) starts to rearrange his pattern to a 'mandala-like' pattern (time: 6:45). A little bit later (time: 7:45), the kindergarten teacher clears up her own placemat and comments: You will have your own ideas to continue. Karl was the only child in this situation, which grasp the idea of symmetry of the Mandala, starting with "the centre" and he worked twenty minutes to finish a very complex, almost symmetric pattern, whereas the three others pursue to produce figurative arrangements and finish the much earlier.

Teacher (7:45)	Karl (7:45)	Karl (27:15)

In general, this situation is formed by very free pattern creations of the children. Several times, the kindergarten teacher ties mathematical oriented comments to the utterances and activities of the children. She does not consist on following her suggestions and the children were not push directly to elaborate their ideas or arrangements to a 'mathematical' one. In particular, she emphasis, that every one has to trial his/her own ideas – and the children did so. The individual constructions where affect by individual examination with the material, in reciprocal awareness, but not in extensive interchanges of ideas. Thus, describing this situation as a 'learning opportunity', constructivist assumption of learning and mind are required, which is the basis for the concept *children as thinkers*. Regarding the concentration on the individual process and the minor emphasis on interpersonal exchange of ideas, radical constructivism as in the instruction model *acquisition* (Rogoff et al.; see above) seems to be the underlying learning concept.

4.3 Butterfly puzzle

In this situation, the kindergarten teacher selects a butterfly puzzle of a set of symmetric puzzles for the setting. Before entering the room with the two children, she prepared the puzzle, putting the shape of the puzzle and coloured wooden triangles (equilateral; red, blue and yellow) in the middle of the table. She starts the situation questioning the children "Do you know what we will do today?" The children uttered some incomprehensible words, thus, the teacher go on:

TE: Let's make a butterfly together. We will colour the aerofoil in a nice way N: I know TE: With which colour you will start	
N: with the blue one[13] TE: we will start together with one (...) Berk will also participate B: with here . I make the red one TE: okay (...) TE: You can put them together in this way (rearranging the triangles of the boys), look	

[13] Both children have not German as mother tongue; mostly, their verbal contribution are grammatical not correct.

Just from the beginning, the children were encouraged to participate in the realisation of a "shared endeavors" (Rogoff et al. 1996, p. 389), where it seems that the kindergarten teacher have an idea of symmetrical puzzle just from the beginning ("we will start together with one" AEROFOIL), but she only implicit inform the children about the idea of colouring the butterfly symmetrical ("we will the butterfly in a nice way"). First, the focus is on the practice of parqueting the area with equilateral triangles. Thus, the teacher regulates the first attempts of them ("you can put them together in this way, look"). Later on, the teacher follows her own colouring ideas, witch seem to be guided by the second part of the whole project: Her triangles supplement the pattern of the two boys in a way, that small plain-coloured areas (e.g. 'lines' of yellow triangles) occur.

After finishing the first aerofoil, she introduces the second part of the project to the children:

TE: look . do you know how a butterfly looks like . . outside, when he is sitting on a flower N: yes B: (*puts a triangle in the second aerofoil*) TE: Berk, wait a moment . this side (*showing the coloured left side*) looks the same as the other one (*showing the empty right side*) Children: (*nodding*) (...)	
TE: oh, look . it starts again . . what did you need now a lot N + B: yellow (*both grasp a yellow triangel*) N: but I will start at the bottom TE: but then you must look . there, we did not need yellow . . what will we need then N: ah, (a lot) red (*both boys take red triangles and start the third line at the bottom*)	

In the second part of the joint problem solving, she refers several times to the plain-coloured areas as orientation for coping the whole pattern. Thereby, the children are free in a certain degree. Thus, the boys decided to start at the bottom of the third line (with a red line) instead with the lot of yellow at the top, as suggested by the kindergarten teacher.

As other forms of managing the realisation of a symmetrical pattern, the kindergarten teacher covers parts of the finished aerofoil and points on triangles in the finished aerofoil, both forms of focussing and ordering the process of problem solving. At the end, they reflect the finished puzzle with two "equal coloured aerofoils".

Thus, this situation can be seen as a type of children as knowledgeable, whereas the children were encouraged and enabled to participate in the 'practice of producing a symmetrical pattern'. Within the joint process of fulfilling the problem, which was set up by the kindergarten teacher, the kindergarten teacher established several forms of ordering and organising the puzzle. She adjusted this forms to the activities of the

children and the children adopted their own ideas to her management process. Thus, on the one hand the children are not as free in their ideas and constructions as in the situation ‘A lot of things’ but on the other hand they are not so strict bounded to the ideas of the kindergarten teacher as in the situation ‘Coloured nails’.

5 FINAL REMARKS

The goal of these everyday educational considerations is not to play the resulting concepts against each other in the sense of good or poor practices. Much more the detection of the respective strengths and weaknesses of individual everyday educational concepts and the determination of the learned or assimilated mathematics with regards to their effects for the situations is the issue. Only in this manner can the existing everyday practices of the teachers be used to change the practices of mathematical education. In this manner, the everyday educational concept of children as knowers often leads to very strict methods of processing the tasks determined by the teacher in small steps and along with that to mathematics composed of prepositional knowledge from guidelines. Of course, skillfulness in the organisation of the learning settings and the motivation of the children for the pre-determined tasks is often demonstrated in these situations. The concept of children as thinkers benefits significantly in comparison with the other ideas about children; mathematics become a creative interaction with the objects provides through which the children have time to explore themselves. However, an outward proclivity for further discovery is often missing from the creations of the children. In the situations that can be classified under the concept of children as knowledgeable, potential mathematical learning momentum can only develop if the teacher wants to focus on the mathematical content of the cultural practice as part of the mutual mastery. Thus, we have observed a similar situation to “butterfly puzzle”, but the kindergarten teacher seems not to anticipate the difficulties by a very mixed pattern, which is to cope. Thus, the whole community-of-learners, including the adult, fails in finishing the symmetrical pattern.

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