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The first Discernment into the NMT-Family (Interactional Niche in the development of mathematical thinking in familial situations)

IDeA Center and Project erStMaL

The IDeA (Center for Research on **I**ndividual **De**velopment and **A**daptive Education of Children at Risk) was founded by the German Institute for International Educational Research (DIPF) and Goethe Universität Frankfurt. [1] The center explores extensively the development of children at risk and the processes of individual learning.

One of research project of IDeA Center is named as Project erStMaL (early Steps in Mathematics Learning), which investigates the mathematical development of children with regard to their migration background. [2] It is planned as a longitudinal study to follow children from the age of three, until the third year of primary school from a socio-constructivist perspective. While the first survey period covers only kindergarten children, the second survey period covers the same children in primary school ages.

A familial Study in the erStMaL Project: erStMaL-FaSt

In the Study erStMaL, it is also performed a family study, which is designed as a longitudinal study and named as erStMaL- FaSt (early Steps in Mathematics Learning-Family Study). It deals with the impact of the familial socialization for mathematics learning.

For erStMaL-FaSt, 12 children who are about at the age of 4 are chosen from a larger sample, which belongs to the project erStMaL. The criteria are the ethnic background (German or Turkish), the duration of the formal education of the parents and the sibling situation within the families. The research design is seen below:

12 Families		with sibling	without sibling
Higher Educational Families	Turkish/ German	1	2
	German	2	1
Lower Educational Families	Turkish/ German	1	2
	German	2	1

Table 1: Research design

Data collection comprises of recorded videos and their transcripts. Once in a year, an appointment is arranged with each family. This leads step by step to a collection of data from each child. The collection of these data for each child is called as "the sample of the child."

[1] Retrieved from http://www.idea-frankfurt.eu/homepage/about-idea

[2] Retrieved from http://www.idea-frankfurt.eu/homepage/idea-projects/projekt-erstmal

In these appointments the erStMaL child is video-recorded together with members of the family while they are playing. For each sample, the following table is designed:

Sample Design	I	erStMaL child as single child	II	erStMaL Child as sibling
erStMaL Child is playing with	Ia	mother or one member of family (e.g. father)	IIa	mother or one member of family (e.g. father) and sibling
	Ib	mother and one member of family (e.g. father)	IIb	mother, sibling and one member of family (e.g. father)

Table 2: Design of a sample during an observation phase

For erStMaL-FaSt, four play situations are conceived, which are constructed according to specific design patterns (see also Vogel & Wippermann 2005; Acar & Krummheuer 2011). They refer to two mathematical domains: geometry and measurement. Each play is categorized according to recording schedules of each year, as seen in table 3:

	1st Year	2nd Year	3rd Year	4th Year
Geometry	Building 01	Building 02	Building 03	Building 04
	Geometric Bodies 01	Geometric Bodies 02	Geometric Bodies 03	Geometric Bodies 04
Measurement	Weight 01	Weight 02	Weight 03	Weight 04
	Towers 01	Towers 02	Towers 03	Towers 04

Table 3: Categorization of Plays

For data collection, an appointment must be arranged with family by giving the flexibility to choose place and time. In appointments, all plays are explained and instruction manuals are given just before the game. Afterwards, the family is left alone to make the members feel themselves comfortable while the video recorders are turned on and recording them. Family is allowed to choose at least 2 games out of 4 and to perform them. Moreover, family members are free to play, in which language they want. In addition, for participation of all families, instruction manuals of each play are made both in German and English. All materials of the games are provided by erStMaL-FaSt team and left in the recording room with family.

Now, I am ready with the second recordings to start setting up the new play situations.

Theoretical Framework of erStMaL-FaSt

Parents are their children's first and continuing "educators" (Mills 2002, P.1). Thus for the learning of mathematics, the family functions are an ongoing "support system", parallel to kindergarten, preschool and (primary) school. By the term "support system", it is referred to the idea of any socio- constructivist theory, which means that the cognitive development of an individual is constitutively bound to the participation of this individual in a variety of social interactions. With respect of Bruner's concept of a Language

Acquisition Support System (LASS), we propose a similar concept for the learning of mathematics, which we call analogically the "Mathematics Learning Support System" (MLSS) (Bruner 1986, p. 77; see also Acar & Krummheuer 2011, Krummheuer 2011b and Tiedemann 2010).

> **"Mathematical support exists of patterns and routines of interaction, which are realized by adult and child, and also, in which the child is supported to participate in a mathematical discourse" (Tiedemann 2010, S.154; translation by Fehler! Verweisquelle konnte nicht gefunden werden.).**

The more children experience mathematical situations in their families, the more learning of mathematics in early years occurs in the different emerging forms of participation in everyday situations in their families (Acar 2011,P.1861). In FaSt, it is focused on only the mathematical play situations. While adults maintain the playfulness of these activities, mathematical competitions , in which occurs a support system emerge there.

For the comparison among the various family situations and for the longitudinal analyses as well we the concept of the "interactional niche in the development of mathematical thinking" (NMT), which has been introduced by Krummheuer 2011, will be used. It also includes the advantage of more closely analyzing the relationship between familial mathematical learning occasions with those, which take place in preschool, kindergarten and/or primary math-classes.

With regard to the design of Family Study, three components of NMT-Family can be shown as it is seen below:

NMT (Family)	**component: content**	**component: cooperation**	**component: pedagogy and education**
aspect of allocation	mathematical domains: "Geometry" and "Measurement"	Play as a familial arrangements for cooperation	developmental theories of mathematics education and proposals of activeness for parents on this theoretical basis
aspect of situation	interactive negotiation of the rules of play and the content	leeway of participation	folk theories of mathematics education, everyday routines in mathematics education; MLSS

Table 4. Structure of developmental niche in familial context

The structure of these three components is detailed and explained below:

1. Content:

In the practice of erStMaL-FaSt, children and their families are confronted with mathematical play situations, which are either in mathematical domain "Geometry" or in mathematical domain "Measurement". The play situations in erStMaL-FaSt are designed according to a specific design, which supposedly gives the families areas and opportunities for interactive negotiations. From the situational perspective, in these play situations, processes of negotiation, in which the rules of play and/or mathematical topics might be thematized, emerge.

2. Cooperation:

The process of cooperation between the adult and child provides the opportunity to refine their thinking and to make their performance more effective. Depending on this cooperation, a different leeway of participation comes forward.

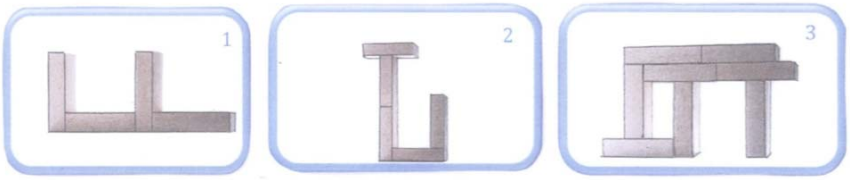
"Leeway of participation" („Partizipationsspielraum", Brandt 2004) is one of the interactionistic approaches, by which a child explores his/her cultural environment while co-constructing it. So, this is a concept belonging to the situational aspect. Brandt (2004) explains that the participants interactively accomplish different margins of leeways of participation that are conducive or restrictive to the mathematical development of a child. (see also Krummheuer, G. 2011c; 2011d). Alongside of contents, the children are involved in the social settings in the play situations, which are variously structured as in child-parents interaction and/or child-sibling interaction. These social settings need to be accomplished in the process of interaction.

3. Pedagogy and Education:

Developmental theories and theories of mathematics education describe and delineate learning paths for the children's mathematical growth from which point of view. With the respect to the folk pedagogy, the participating adults and children become situationally active and operant in the concrete interaction. Thus, in this system MLSS occurs in different ways.

A Play: Building 01

The mathematical play "Building 01" refers to geometry and spatial thinking. The family is supposed to build three-dimensional version of the picture with wooden bricks, which are in the same size and weight, by looking from the game card, which shows the structure as two-dimensional version. Supposedly, they perform the relations between two- and three-dimensional representations. The player chooses one card from the deck and builds the image on the card with wooden bricks. In the play, cards are placed on the table face down. Each card has a different difficulty level from 1 to 3. The cards with the number 1 are the easiest and the cards with the number 3 are the hardest.



Picture 1: the game cards in different levels

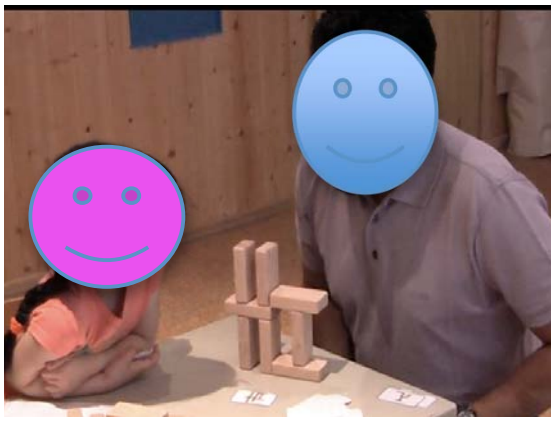
An Example: Family Kil

The required information about Family Kil is in the following table:

Father Ayse Mother 	Ayse (erStMaL-Child)	6;7 years old German, Turkish rudimental
	Mother	Studied 10 years German, Turkish rudimental Higher Education
	Father	Studied 13 years German, Turkish rudimental Higher Education

Table 5: Family Kil

In chosen game, Ayse and her father play together in the absence of her mother. In total, they play 14 rounds by turns.



Picture 2: Ayse and her father during the play

Up till the chosen and transcribed scene, they have played just 2 rounds as it is seen on the table 8. Mr. Kil started to play. Till 3rd round, they built each cards correct, so that Mr. Kil got 4 Points and Ayse got 3 points.

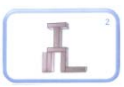
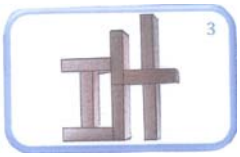
Rounds	Father (Mr.Kil)	Ayse
1. Round		
2. Round		

Table 6: Chosen cards of Ayse and her father in first two rounds

In the 3rd round, Mr. Kil chooses another card and starts to build it.



Picture 3: A card chosen by Mr. Kil in 3rd round

After a while, he completes the assignment as it is seen below:

Picture 4: A built Structure by Mr. Kil

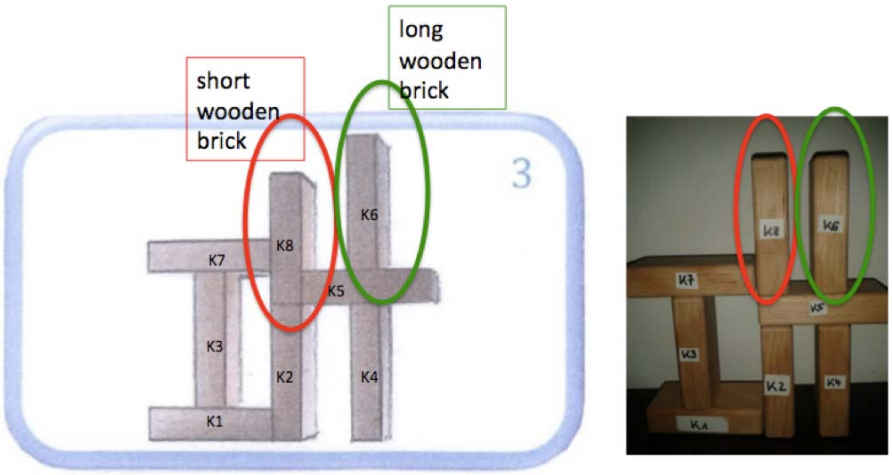
In the chosen Scene, he asks his daughter if the built structure is identical with the figure on the card.

01			Father	right?
02			Ayse	yes.
03			Father	cool.
04			Ayse	but one long and one short. *looks at the card*
05			Father	yes I think the picture is false. *bends to front and*
06				*shows the card with his index finger*
07			Ayse	mmh
08			Father	just look. this is a short block or a short piece of
09				wood.
10			Ayse	*looks at the residuary pile of wooden blocks*
11	04:30		Father	Building block.
12			Ayse	*leans back, looks still at the pile of wooden blocks*
13			Father	*shows the pile of wooden blocks* but there are no
14				short blocks. I think the picture is false. I would
15				say; it is all right. O.K?? *puts the card*
16				*in his left hand and piles it on his other cards*
17				you too?
18			Ayse	*pushes her fathers building with her right hand*
19				mhhm
20		>	Father	good. *bowls over the his building and pushes the*
21				*blocks to the other pile*
22		>	Ayse	*picks a new card with her right hand*

23				*turns the card face, looks at it, then looks at her*
24				*father and laughs*
25			Father	yes.
26			Ayse	mh
27			Father	not so difficult. *scratches his face with his left*
28				*hand*
29			Ayse	*She picks up the card and puts it on the table*
30				*so that she looks on it like in line 22*
31				*She separates the cards from the other cards*
32				*which are used in exercises they have before.*
33				but..one more.. a long one. heh? *looks at the pile of*
34				*blocks on the table*
35			Father	No no it looks like that. *takes her cards away*

By posing this question, Mr. Kil might try to test Asye's spatial structuring, if she has mentally the same structure in her mind. As a reply, Ayse gives him a positive feedback. Thus, at the first sight, Ayse reacts that her father built the picture correctly as it is seen on the card. <01-02>

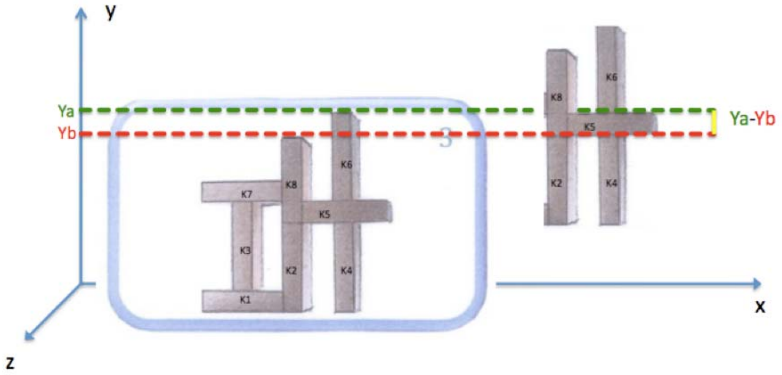
But after she checks the card, she realizes the discrepancy between the constructed building and picture. but one long and one short.<04>, she says. Actually she doesn't depict exactly, which wooden bricks, she meant, are long and short. When the card is compared with the structure of the building, it could be interpreted that she could mean the following:



Picture 5: Comparison between the figure on the chosen card and the built structure

As it's seen in picture 5, there is a difference between the figure on the chosen card and the built structure. According to the visual discrimination, it can be said that Ayse can represent wooden bricks at the detailed level of shapes (Clements & Samara 2007, P.511). Topologically, she can also coordinate simply both structures and realize that the wooden sticks, K8 and K6, are not horizontally in the same height.
As a reply, her father approves her critique and bases his "discrepancy" on the chosen card <05-09>. He might think that the figure on the card is less stable and would collapse easily, whereas the construction they build seems to be quite firm; especially the cross consisting of K4, K5,and K6 is statically better integrated into the entire building. Another

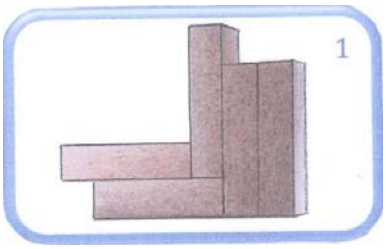
option is that he sees the difference but that he judges that the difference between picture and construction is less important in relation to what his daughter has achieved. He goes on explaining the fault on the card, that one building block, most probably K8, is scratched shorter than others on the card. It also could be that also he does not see the "discrepancy" in their construction.



Picture 6: The jut due to K5 in coordinate axis

With regard to his argument, Ayse looks up to the pile of wooden bricks as she checks out whether the wooden bricks are in the same length and size <10-12>. Her father, however, repeats his statement as follow: Although there are not short wooden bricks, there is a short wooden brick on the chosen card. Hence, the card is false <13-14>. Then he insists that he builds the figure correctly and asks for Ayse's approval<14-17>. She doesn't go on her argument and doesn't make any other commentary, so her father behaves like she doesn't reject his argument <18-21>. Without remarking the effect of K5 (Ya-Yb), the discussion leads to somehow ambiguous consensus (see picture 6).

Then Ayse selects a new card from the deck. Hereby Mr. Kil's turn in 3rd round ends up and Ayse's turn begins.

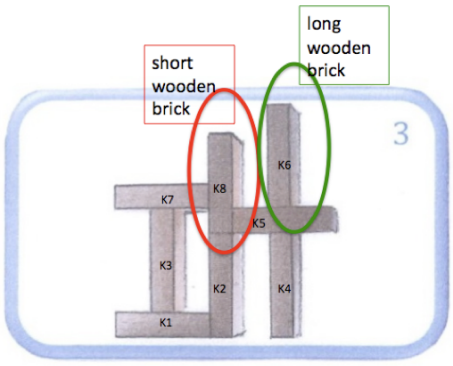


Picture 7: A card chosen by Ayse in 3rd round

She laughs at her father, after she looks at the card <22-24>. Her reaction could be interpreted, that she asks her father for help to "construct" it, because it is appears her as to difficult to do it by herself. The father might understand her this way but tells her, that she thinks that the figure on the card is not difficult <25-28>.

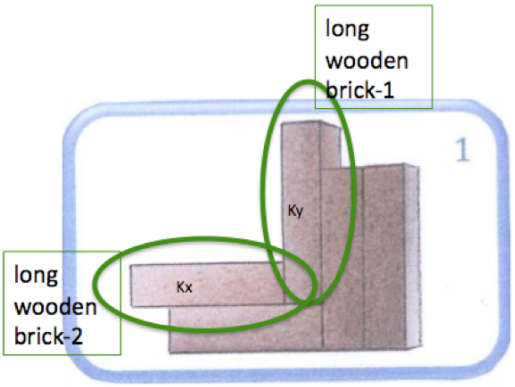
She takes upright position to build the card. But after she checks the card, she remarks that, she needs one more long wooden brick <29-35>. In her statement, she uses an adjective "one more", by mentioning this, she means that she has already one, but she needs one wooden brick more. There are two mentions in this sentence:

1. In her father's turn, she sees "one long" wooden brick (see Picture 8). That's why, she wants "one more" while she "has already one".



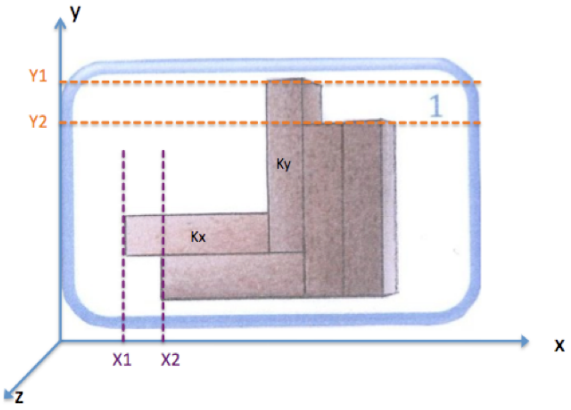
Picture 8: the long wooden brick on Mr.Kil's card (K6)

2. She needs one more "long wooden brick" because; there are "two long wooden bricks" in the image on her card. (see Picture 9)



Picture 9: long wooden bricks on Ayse's card (Kx and Ky)

Here I see a high attentiveness of Ayse, that she can realize the juts of Kx and Ky.



Picture 10: Juts of Kx and Ky in coordinate axis

Actually, she repeats and defends her arguments, what she already mentions during her father's turn. Because, her father cannot give a persuasive explanation why the lengths of wooden bricks are seen different in the image on his card. She cannot relinquish her argument that she needs two long wooden bricks to build the figure on her card.

Despite all her claims, her father only answers that the figure on the card looks like that<36>. Obviously, he gives her no further explanations. Hereby with this explanation, a situation, in which two similar problems are expressed exactly as two contrary positions, emerges. Although he says that the card is false in his turn, in this situation he rationalizes it by saying that the card is like as it is seen. He is always such in a case that he ties in his argument with the card. In his turn, the level of difficulty of chosen card was three and now in Ayse's turn, a card with the lowest level of difficulty is chosen. That means, while Mr.Kil has the most difficult card, Ayse has the easiest. Thus, it is remarkable to see the difference of the constructions on both cards.

I benefit to say that although Ayse abstains from building this card, Mr Kil encourags her to build it and at the end of 3rd turn, she correctly builds the chosen card.

As a summary, in this scene, although there is a conflict among Mr.Kil's arguments, there occurs a developmental niche for Ayse. According to Cross, Woods and Schweingruber 's work (2009, P. 187), which is about the spatial development of children, this conflict reinforces Ayse's spatial thinking. Hereby her spatial structuring and visual discrimination are enhanced as realizing the discrepancy between buildings and pictures and building a figure, which has juts, without using long wooden bricks.

According to Clements & Samara, 5-years-old children can metrically represent spatial information in a polar coordinate task, using the same two dimensions as adults, radius and angle (2007, P. 498). If this argument would be disregarded, it could be assumed that Ayse's spatial thinking is not developed enough to realize 2- and 3-dimensional coordinate axis. In any case, a developmental niche for Ayse due to the sensitizing between the building and the picture occurs there. In her father's turn, she sees the discrepancy between building and picture. Thus, in her turn, she acts already on the base of a sensitization for the overlapped blocks. Although she abstains from building the card and she is cognitively in the peradventure circumstance, her father encourags her to carry on building. This encouragement is also another sensitization for Ayse, that she can do something despite her assumed difficulties. She focuses on the problem and solves it. Although there is no long wooden brick, she builds the overlapped figure. Thus, in this case, a development niche for Ayse has occurred.

This whole analysis can be structured according to the three components of an interactional developmental niche in familial context.

1. Component "Content":

Block Building provides a view of children's initial abilities to compose 3-D objects. In the chosen play ,three goals are pursued;
1. Spatial structuring,
2. Operating shapes and figures
3. Static balance between wooden bricks and,
4. Coping with the different difficulty levels on the cards.

As Clements & Samara (2007, P.494) points out spatial relations require attention, Ayse sensitizes and realizes the spatial relations between 2- and 3-dimensional objects.

Cross, Woods and Schweingruber (National Research Council, Committee on Early Childhood) also report that (2009, p.191):

> **"5-years-old children can understand and can replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the relating parts and wholes level."**

Thus, she can relate parts and wholes between two- and three-dimensional structures.

2. Component "cooperation":

Verbal interaction is very important and parental scaffolding of spatial communication develops children's perform (see also Clements & Samara 2007).
The play situation is constant and it is directed by father. In his turn, he allocates to Ayse a limited leeway of participation. But, in her turn, he allocates an opened up leeway of participation for Ayse, that she should believe in herself and overcome the difficulties. Thus, in this play situation occur different leeways of participation.

3. Component "pedagogy and education":

The chosen play situation is constructed along a uniform didactical design pattern and refers to the spatial structuring in geometry.

> "**Spatial structuring is the mental operation of constructing an organization or form for an object or set of objects in space"** (Clements & Samara 2007, 498).

In the chosen scene, the opportunities to sensitize 2- and 3- dimensional bodies (objects) have occurred. In this way, the graphical- and spatial-development are strongly assisted.

> **"The play, for the child and for the adult alike, is a way of using mind, or better yet, an attitude toward the use of mind"**(Bruner 1983, p. 69).

Her fathers' "discrepancy" and arguments help her recognize feasibility of the building juts without longs wooden bricks. With the respect of Folk theory, his encouragement provides Ayse the success of learning 2- and 3-dimensional coordinate axis. She abstains from building this card but her father encourages her to build it. Therefore, she overcomes the difficulty and builds the figure on the as it is seen.

These insights can be assembled in the NMT-Family table as follows:

NMT (Family Kil) Building 01	**component: content**	**component: cooperation**	**component: pedagogy and education**
aspect of allocation	Geometry, operating with shapes and figures, spatial structuring Static balance between wooden bricks	Playing with father	the development of spatial skills and transformational abilities in spatial structuring
situation	discrepancy between the solutions, ambiguous consensus	different leeways of participation	The father's "discrepancy" and arguments assist Ayse to recognize feasibility of the building juts without long wooden bricks. His encouragement provides Ayse the success of learning 2- and 3-dimensional coordinate axis and overcoming the difficulty mentioned before.

Table 7: NMT-Family Kil

Afterword / Conclusion

> **"The play under the control of the player gives to the child his first and most crucial opportunity to have the courage to think, to talk, and perhaps even to be himself "** (Bruner 1983, p. 69).

On this account, it seems necessary that a vivid interaction in play situations arises there. For the interactive creation of learning opportunities, it is obviously not commendatory that these interactions lead to a consensus among the participants. The chosen play situation "Building 01" includes opportunities to negotiate interactively (see Miller 1986, P.176; Acar&Krummheuer 2011, P. 168).
Krummheuer underlines the importance of interactional phenomenon on the spatial thinking:

> **"The essential impact supporting the development of spatial thinking seems to be based on an interactional phenomenon in which the participants create a communicative atmosphere, in which they at least can explicitly disagree about the construction of the buildings" (2011d, P.17).**

In chosen scene, there is an "antagonisms" among the father's explanations, although there is no consensus between father and his daughter about the first solution.

> **"Spatial thinking is contrasted with an alternative mode of information processing language-based, drawing on analytical, logical-deductive reasoning"** (Kersh, Casey & Young 2008, P. 234).

In this sense, due to Mr.Kil's oversights or his insufficient informations, a developmental niche for Ayse on her spatial skills emerges there. Her spatial thinking is contrasted with her and her father's logical-deductive reasoning.

Contrary to Ayse's arguments, the father always offers another reasoning. Herewith, she experiences the possibilities and learns to associate two- and three-dimensional coordinate axis. On the other hand the language-based encouragement of her father provides the development of her spatial thinking as well as her personal/social skills as mentioned before. Kersh, Casey & Young emphasize the reflection of block play on the social skills:

> "**Engaging in block play helps children acquire a diverse range of valuable competencies and knowledge, from social skills to the foundations for later math achievement"** (2008, P. 237).

Cross, Woods and Schweingruber (National Research Council, Committee on Early Childhood)(2009, p.187) report that 5-year-old children can understand and replicate the perspective of a different viewer. They point out that 5-year-old children can understand the substitution of shapes and can build complex structures. They proceed:

> **"They can build structures with cubes or building blocks from 2D pictures of these structures. Children of this age also can learn to move squares and right triangles on grids to create original designs. They can also record these designs on squared-grid paper."** (2009, P. 191).

With the regard of this explanation, it is seen that at this age (about 6 years old) children are able to structure two- and three-dimensional structures according to coordinates. My first insight to reveal this tendency is firstly parents' assignment, regardless of whether they have good spatial skills or enough geometrical knowledge.

In another long-term analysis of the development of spatial skills in familial context based on the notion of NMT-Family, I could declare that this development occurs slightly independent from parent's geometrical knowledge. In the chosen example, with the father's encourage, Ayse correctly builds the figure at the end. This is also an overcareful learning progress, in which the interactional developmental niche occurs and Ayse lives through it.

Clements & Samara stress the spatial processing of children and adults:

> **" Spatial processing in young children is not qualitatively different from that of older children or adults. However, with the age, children produce progressively more elaborate constructions"(2007,P. 512).**

In this sense, I will augment the examples in next two years. It will be excited to find out how NMT-Family functions work on children's spatial development in familial context.

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