Theoretical Fundaments for an Intelligent Tutorial System Towards the Learning of Geometry at a High School Level

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Abstract

This paper aims at laying down the multidisciplinary fundaments of the geogebraTUTOR system (GGBT), a research and technological realisation project developed in didactics of mathematics (mathematics education) jointly with informatics computer science. In its design, GGBT presents as an intelligent tutorial system, which supports the student in the solving of complex problems by assuring the management of discursive messages as well as the management of problems. By situating the learning model upstream and the diagnostic model downstream, GGBT proposes to act on the development of mathematical competencies by offering a control of the acquiring of knowledge in the interaction between the student and the milieu, which allows for the adaptation of the instructional model (learning opportunities) according to the instrumented actions of the student. The notions of inferential and construction graph, which reveals themselves as a structured bridge (interface) between the very contextualised world of didactical contracts and the formal computer science models, structures GGBT in a way to allow the tutorial action to adjust itself to the competential habits conveyed by a certain classroom of students and to be enriched by the research results in mathematical education.

Keywords

Mathematical education (didactics of mathematics) • Intelligent tutorial system • Mathematical competencies • Computer science models (informatics) • Geometry learning

1 Preliminary Point of View on the Tutorial Systems for the Learning of Geometry

Geometry at a High school level can be seen as a deductive science allowing the solving of problems in the mathematical field as well as a theoretical reference, which orients the wider process of extra mathematical modelling, this permitting amongst other things the laying down of problems inspired by what is referred to as the real world or reality. Even if the solving of modelling or proof problems presents, along side of the curricular obligations, as a mathematical competence to prioritize in the education of young people, it remains difficult to develop in regards of the traditional relationship between the teacher who, alone in front of his class, looks to give insight into the mathematical reasoning, calculations and other problems to which are confronted students. Also, when it's the instrumental workings of the dynamic geometry tools that liven up his didactic interventions, the teacher may feel at loss when faced with the flow of interactions between each student and the computer device, and this in spite of the fact that these interactions constitute a source of insight into the evolution and progression of mathematical competencies and knowledge.

The idea of a tutorial system accompanying the student in the solving of problems and thus completing the work of the teacher is undoubtedly not new. Amongst the recent technological realisations that relate best to **geogebraTUTOR** (Richard & al., 2007), more precisely to the current notion of our tutorial system, proper to mention to begin with the **Advanced Geometry Tutor** (Matsuda & Vanlehn, 2005), the **Baghera** project (Laboratoire Leibniz, 2003), the **Cabri-Euclide** microworld (Luengo, 2005) and the **Geometry Explanation Tutor** (Aleven & al., 2002). All these systems essentially establish themselves accordingly to formal geometry models that, in spite of clear IT programming advantages, imply supporting an axiomatic approach for the student's development of geometric competencies. There is also the **Andes Physics Tutor** (Vanlehn et al. 2005). Although this system is dedicated to the teaching of physics, it proposes to the student different means of analytical modelling (translation into equation or function) in the context of problem situations, some of which are pre modelled in geometry. However, isn't dynamic geometry a kind of «first physics» which preserves the construction's logic (balance of the forces) in movement or in the exploration of particular cases? Finally, there is **AgentGeom** (Cobo & al., 2007) and **Turing** (Richard & al., 2007), systems anterior to geogebraTUTOR (GGBT). Contrarily to previous systems, these devices are essentially based on cognitive geometry models, AgentGeom's tutorial action occuring mostly during the geometrical shape construction and AgentGeom's occurs while the solving of a problem takes place.

2 Research Problem of GeogebraTUTOR: Student-Milieu Interactions and IT Questions

Stemming from a multidisciplinary project between didactics of mathematics and computer science, GGBT defines itself as an Intelligent Tutorial System (ITS) that supports the student in the solving of complex problems[1] by assuring the management of the discursive messages as well as the management of the problems. In regards to the conception, our ITS poses a simulated didactic relationship in which the tutorial agent plays, in spite of an individualised personality according to the Iterative Learner's Model (ILM), a teacher's role which is complementary to the one of the regular teacher. This means, from the point of view of the theory of didactical situations in mathematics by Brousseau (1998), that the main act of the teacher (relationship 1 and 2 in Diagram 1) is transposed into the «student-milieu» system (relationship 6 and 7, ibidem). Consequently, the notion of milieu needs a new distinction: the «didactic milieu» is antagonistic to the taught system (original definition of the author), in which the tutor agent appears as a sub-system, and the «virtual milieu», which competes with the student with whom it negotiates, thus establishing, conjointly, the main act of the tutor agent (relationship 6 on 7).

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[1] By *complex problems* we mean the existence of many solving processes (heuristic requirement), the mobilization of a network of mathematical concepts and processes (cognitive requirement), the existence of an argumentative approach, of a multi-stage reasoning or non-routine calculations (discursive requirements) and the development of groups of competencies that go beyond simple reproduction (competential requirement).

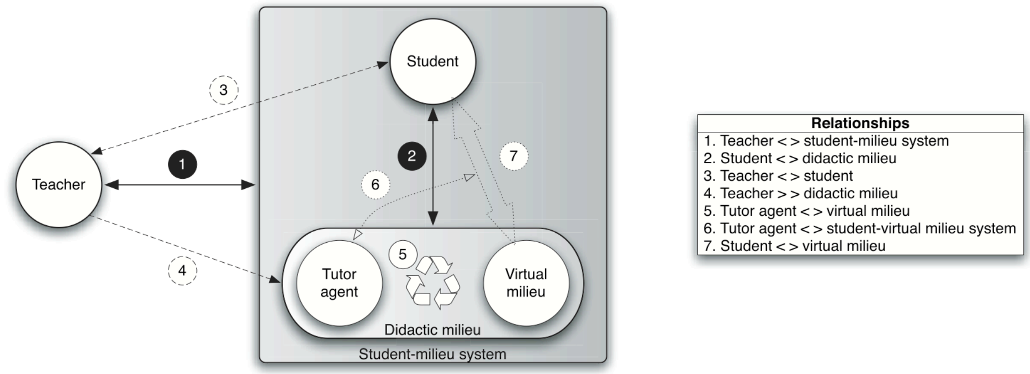

Diagram 1. Situational map.

Regarding the functioning of the system, the student constructs shapes, writes discursive propositions or calls upon mathematical properties in the solving of a root problem situation. The tutor agent returns to the student a discursive message based on his significant actions and, when the student stalls, may propose a related sub-problem to boost the initial solving process. We name the sub-problems *cognitive messages*, by analogy to the approach of Carnegie Learning's Cognitive Tutor[2]. However, in opposition to the approach by sub-problems equivalent to parts of the root problem, our cognitive messages are related to it thematically according to Neighbourhood criteria and anticipated difficulty levels. If the management of the discursive messages raises the IT issue of the recognition of the student's reasoning process from his significant actions, the management of the cognitive messages raises the IT issue of the conceptual or procedural, heuristical, semiotic or metamathematical changes during the solving process (rupture point). Finally, the management of the problems raises the IT issue of the recognition of the similarity between problems while avoiding, by means of all the messages, to give at the same time answers that modify considerably the stakes in the devolution of the root problem.

3 Research and Development Approach

3.1 Merger of two Paradigms from the Point of View of Didactics Epistemology

The implementation of geogebraTUTOR is part of a research and developments project that begun with the AgentGeom and Turing projects: it borrows its conceptual melting pot and its methodological approach. In other terms, the learning models we claim to draw with the usage of our ITS are essentially based on the Theory of Didactical Situations in Mathematics (TDS), the Duval's Theory of Language Functions (TLF) and the Instrumentation Theory by Rabardel (IT), while conjugating The Grounded Theory Analysis by Glaser and Strauss (GTA) and the Didactics Engineering by Artigue (DE) as source methods. Therefore, with our prior projects, we verified the extension of the TDS

[2] See <http://www.carnegielearning.com>.

+ TLF + IT theoretical framework when these theories are applied to an ITS for the learning of geometry at a high school level. However, the hypothetical-deductive paradigm, which had then allowed us to validate separately our research hypotheses with regard to confined situations has revealed itself to be insufficient in a perspective of integration AgentGeom + Turing, which allows the dynamic evolution of the student-milieu interactions. That is why the emergence of learning models adapted to the usage of GGBT requires to add a comparative-inductive paradigm to our current approach, a paradigm we propose on the basis of an GTA + DE integration.

Without going into the details of the methodological integration, it's suitable to stress the fact that our approach differs from regular experimental methods, in the field of science of education, by its validation mode. Accordingly to the method favoured by the DE, this validation mode is internal and is based on the comparison between an a priori analysis, which relies on certain hypotheses, and an a posteriori analysis of the visible and significant student-milieu interactions. In other words, there are no control groups or comparison between students using GGBT or non-users. In addition, these interactions consider the integration of implicit models, among which the mathematical models implemented in the IT device, like those that underline the interface or the computations, added to the implicit key aspects of the instructional design or of specific didactical contracts (in regards of relationships 1 and 6 of Diagram 1). Finally, this validation mode is compatible with the idea of technological transfer into research in the fields of didactics of mathematics and ITS development, since it allows the consideration of student behaviours observed during the use of GGBT, not only for the sake of the student but also in order to progressively improve the intelligent tutor, so endowing the system of an intelligence resulting form the convergence of successive a priori and a posteriori analyses.

3.2 Student's Interface and the System's Structure

From a computer science point of view, it is important to propose to the user an interface paradigm with which he is familiar to facilitate the instrumental genesis or to mechanize, in a certain way, the schemes of use (in the sense of Rabardel, 1995). The interface we propose to the student is composed of three distinct areas (Illustration 2). At the top of the window, under the contextual menus, can be found the problem statement to which we can link a drawing. These elements are continuously shown and can't be modified by the student. In the middle, the interface of GeoGebra (GGB), without modifications, is found. Besides the known potential of this dynamic geometry software, are integrated advanced Computer Algebra System (CAS) functionalities. Although only a pre-version of these is currently available, it will be possible for the student and the student to use them for the solving of certain designated problems. The GGB interface can propose an initial shape, matching or differing from the drawing linked to the problem statement, on which the student is susceptible of launching his work (construction, movement, etc.). It is also possible to use the input field to enter commands. The tutor agent follows closely every action of the student in the GGB window and tries to identify what seems to be the student's solving strategy, which then allows the system to assist or advise the student. Finally, under the GGB module, we exploit the paradigm of a chat window to simulate a dialogue between the student and the ITS. This one is disposed to respond to the student's actions by a message that appears at the bottom of the window. The actions can be discursive (lower module), graphic or symbolic (GGB module), while the messages of the tutor agent can be discursive (propositions in the lower module) or cognitive (hyperlink towards a sub-problem, see paragraph *Identity Cards and Neighbourhood of the Problem*). Like in regular chat systems, the student (and the system) has access to previous messages.

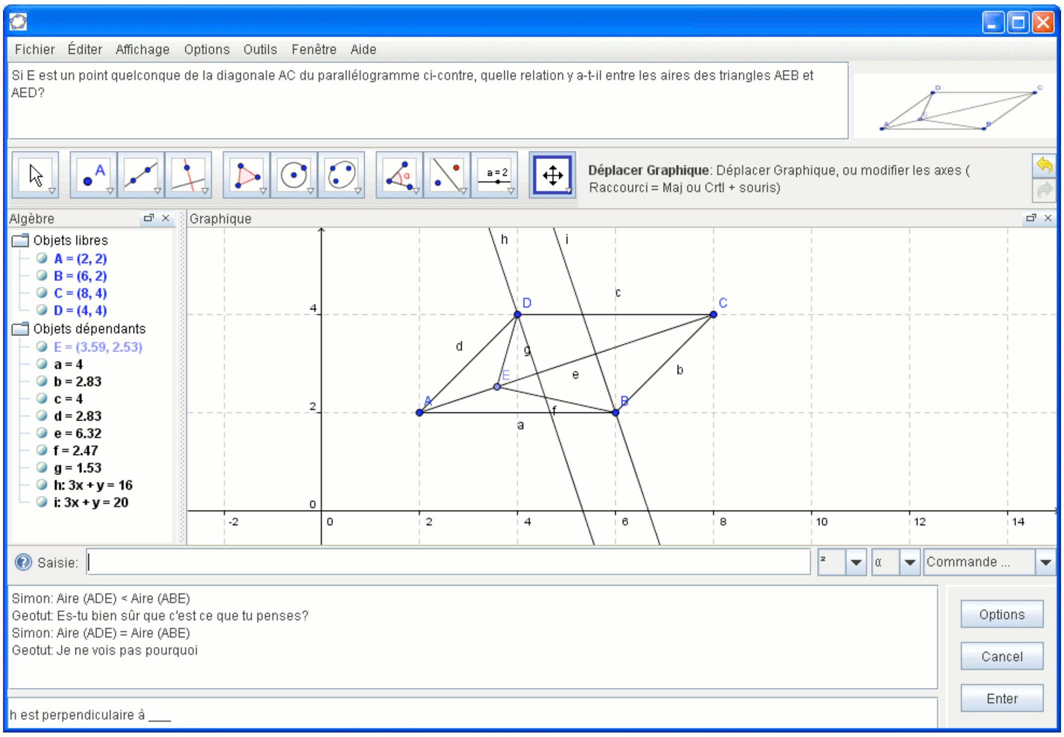

Illustration 2. Student's interface.

Diagram 3 illustrates the internal part of the system. Our ITS is composed of two main subsystems containing six elements each: the GGBT on the left and the tutor agent on the right. This partition aims at granting independence from the student's interface to our tutor agent, to facilitate, among other things, the test and integration of the tutor with the different systems. Ultimately, the tutor could run on a server and exchange, through a network, messages with the interface or even, for validation purposes, be momentarily replaced by a human tutor. However, for the moment, the system is a standalone Java application, which requires no network besides for the downloading of the application.

The GGBT subsystem essentially contains the modules that are related to the user's interface. The GGBT and GGB modules contain the code for the interface. The GGB module supports a computer algebra system (CAS) and an Automatic Deduction system (AD) that can be used by the ITS to execute some verifications on the student's input. Furthermore, this module generates an event log, which can possibly be analysed for the interpretation of the student-milieu interactions. To this are added the graphic and discursive analysis, which allow the transformation of raw actions at the interface into significant actions for the system. In other words, the tutor can treat the student's input as HPDIC messages (acronym of Hypotheses, Propositions, Definitions, Intermediate results and Conclusions). The valid messages for the tutor are stored in the HPDIC database.

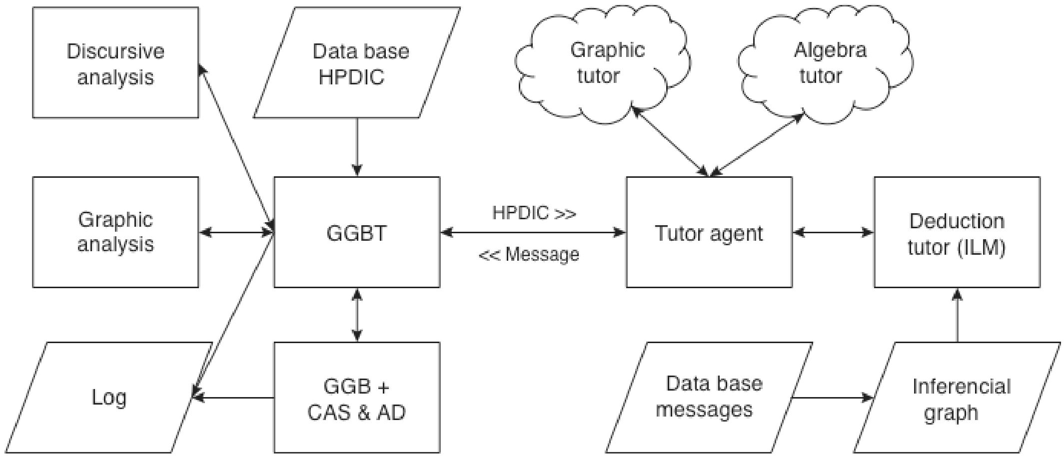

Diagram 3. System's structure.

Once the processing of the student's actions is over, a HPDIC message is sent to the tutor subsystem. The tutor is a multi-agent module that aspires to help the student throughout the resolution, from problem solving to composition, i.e. the writing of a solution (see Diagram 4). If the tutor receives a graphic message, it asks for help from the «graphic tutor» module and it can returns a message to the student's interface. The graphic tutor essentially uses the AgentGeom approach. In the case where the student writes an equation, the «algebra tutor» module is called. This last one is still at its conception phase since it depends on the recent development of the GGB CAS module. When the student enters a discursive proposition at the interface of GGBT, we consider that he is explicitly trying to produce an inference in order to generate a proof. When this occurs, the «deduction tutor» is required. Based on the Turing approach, this tutor contains the inferential graph as well as a bank of discursive messages that could be used to assist the student. The output of the discursive messages is based on the ILM, which is defined as an emergent support model, characteristic of GGBT, which creates the illusion that a communication is taking place with the student (see section *Intelligence of the System and Iterative Learner's Model*). Finally, the tutor agent can reach a state in which the message addresses different aspects of the problem, where help from more than one tutor at a time is necessary (intersection of AgentGeom and Turing in Diagram 4). This way, one construction step may require interpretation by the graphic and deductive tutors, when this step refers to a deduction defined in the inferential graph, for example. In this case, the tutor must decide what message is more useful to the student.

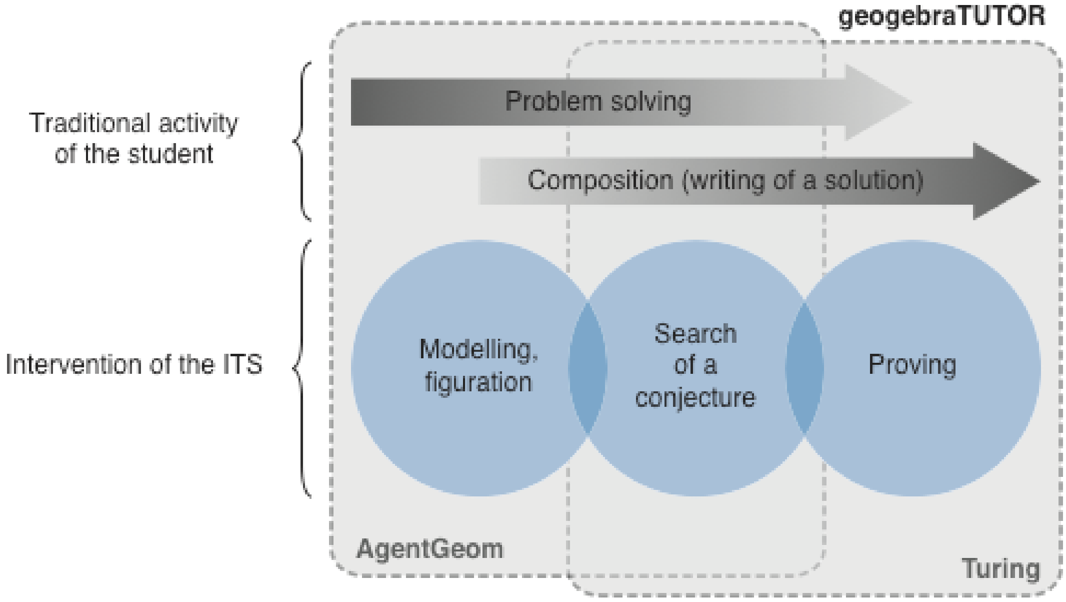

Diagram 4. GGBT's rendering with regards to the steps of the student.

3.3 Reasoning, Graphs and Inferences

Even if we suppose that in their schooling students must one day adhere to pre-existent mathematics (an expert mathematical model), the stakes related to the development of their mathematical competencies result in a progressive learning and mostly very contextualised. In other words, the particularity of the didactical contract in which the students evolve or even the specificity of each problem situation seem to make difficult the practice within an expert model. One of the reasons that lead us to the idea of cognitive geometry lies in our commitment to introduce these contextual aspects into the development of our ITS. Since cognitive geometry can be locally coherent without necessarily being globally coherent – contrarily to formal geometry –, we adopted a structural approach to reasoning, which not only combines these aspects of geometry, but first and foremost allows the IT programming.

Inspired at first of Duval (1995) and Richard's (2004a) works, the concept of inference expressed by dialogue, figural representations and instrumented action, considered as a voluntary reasoning step, intervenes at the heart of our system. Each inference respects an «antecedent → consequent» structure with a justification that «controls» the reasoning step, allowing it or participating in it in order to allow the student to produce a sequence of inferences that remain coherent according to the logic of the problem or significant in its context. The question of significance that is here introduced doesn't aim only at satisfying the possibilities of cognitive geometry (ex: acceptance of certain discursive inferences, in the sense of Duval (1995), or of inferential shortcuts depending on the habits of a didactical contract), but also to deal with the inductive nature of inferences allowed by the practice of dynamic geometry or by the modelling activity. To illustrate our idea, we introduce three types of inferences in the Table 5.

Table 5. Characterisation of the inference types treated by geogebraTUTOR in regards to the control of the justification in the interactions student-milieu, the taking charge of the

inferences by the system and the discursive-graphic effect of these on the student's reasoning.

Typical inference	Control of the justification	Taking charge by the ITS	Discursive-graphic effect
Since « $(PM) \quad (OA)$ » by hypothesis, so, according to «the Thales' Theorem», « $\frac{PM}{4} = \frac{3 - OM}{3}$ ».	Managed by the student	Comparison with the inferential graph	Transformation of states recognised by the student, the consequent is obtained by deduction (cognitive or formal)
Confrontation between a satisfying configuration and a non-satisfying configuration by dynamism of the shape or by construction of it, letting emerge «maximum area is obtained when P is the middle of [AB]».	Shared management by the instantaneous actions-feedbacks	Comparison to the collection of construction graph and to the inferential graphs related to the presumed model	Transformation of significant actions into a state by induction, the consequent is obtained by interpretation of the satisfying configuration (figural inference)
Since « $PMNO = 4 \cdot OM - \frac{4 \cdot OM^2}{3}$ », so, according to « $fMax\left(4 \cdot x - \frac{4 \cdot x^2}{3}, x\right)$ », «the maximum area is obtained when $OM = \frac{3}{2}$ ».	Chosen by the student but essentially managed by the milieu	Use of the CAS or of the GeoGebra oracle, formal comparison of symbolic expressions	Transformation of states by CAS function or oracle, the consequent is produced by the system or chosen by the student according to a list of possibilities («fill the blank» expressions)

At first, for each problem situation, we associate construction graphs (reference shapes) and for each construction graph, an inferential graph. Each construction constitutes a modelling of the problem, which summarises the figural result of a satisfying configuration. These configurations are a result of the basic space of the problem (Cobo & Fortuny, 2000), meaning they origin from the gathering of the solving strategies that can be collected in a given classroom by a teacher or an expert. This way, with the AgentGeom approach, we can compare the student's actions to the collection of satisfying configurations in order to anticipate the model in which the student seems to be evolving. Then, for each satisfying configuration, the correspondent inferential graph must be formed. This graph, inspired of the Turing approach, reveals the different proof strategies that follow from the reading of a satisfying configuration in the shape of a series of well-structured inferences from discursive, symbolic and figural propositions (in the sense of Richard, 2004b).

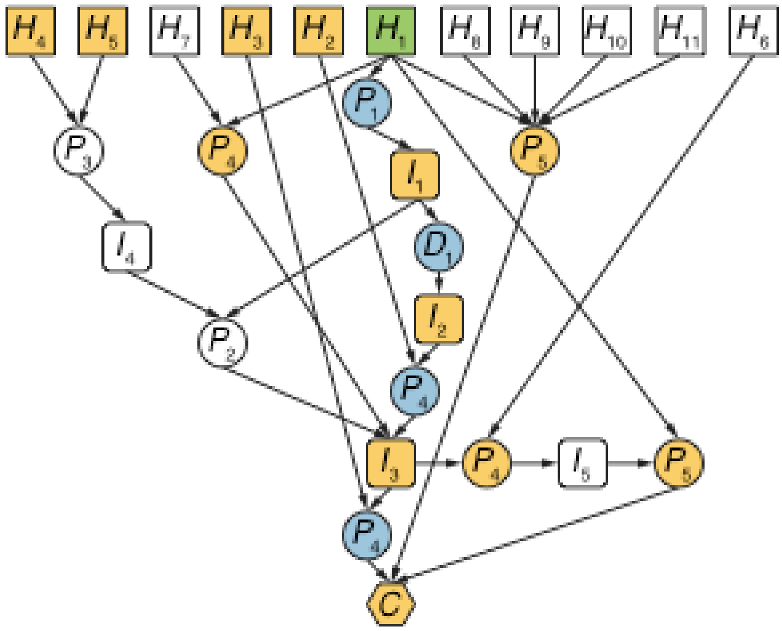

Figure 6. Inferential graph AND/OR.

3.4 Intelligence of the System and Iterative Learner's Model

For the IT system to be able to state the student's stage of advancement in his solution, we have coded the inferential graphs as seen at Figure 6. In these graphs, the nodes P and D[3] are inference justifications, while the nodes prior to them (H or I) or posterior (I or C) are respectively antecedents or consequents of the justifications, the I node playing a double role depending on the considered justification. From a logical point of view, the entrances of each justification node are conjunctions, while all the other links are disjunctions. This means that all the antecedents and a justification are needed to legitimise a consequent. However, there may be many paths to legitimise a consequent, but only one is necessary to complete a proof, which can be considered complete when we obtain a path joining the hypotheses to the conclusion and when all antecedents of the justifications that appears in the path have been activated by the student's actions.

When the student tries to solve a problem, he activates different nodes in the inferential graph. Nevertheless, since the nodes will generally not be activated in a purely descendent or ascendant order, as is presumed in the Matsuda & VanLehn (2003) approach, the ILM supports the student by using the historic of his significant actions, respecting the heuristic aspects of his solving method. Theoretically, the system tries to develop a path around the last action of the student, but if he happens to be stuck, the ILM is lead to relaunch the solving process according to a previous significant action. Consequently, our objective is not to force the student into a deterministic mould, but rather to incite him to follow his intuition or his solution until he obtains a complete solution that can be structured in the logic of the problem.

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[3] The distinction between P and D is on the status of the node (property versus definition) and not of their function in the inference. In fact, the definitions are always logical equivalences, while the properties are implications whose reciprocal logic is not necessarily true.

3.5 Identity Cards and Neighbourhood of the Problems

As in an ordinary class, the organization of the problems is an important question for the development of mathematical competencies. In the classic contract, the teacher gives to the whole class the same group of problems to solve, these generally growing increasingly difficult. However, the stakes of the problems undergo little adaptation according to the evolution of the competencies of each student and even less to their instrumented behaviour. In the section addressing the problematic of GGBT, we introduced the notion of cognitive message; we also evoked the question of neighbourhood of the problems. Really, the criteria for neighbourhood is based on a prerequisite characterization of each problem, which we call identity card, from the categories «processes and concepts», «heuristics», «semiotic» or «metamathematical». Without going into details, we can stress that each category develops according to the shapes or values made possible by the realization of a given didactical contract. Two problems are then neighbours if they share a same subset of values. The information supplied by the identity cards, according to the instrumented behaviours of the students, is susceptible to lead to learning itineraries adapted to the whole class.

The identity cards constitute a global description of the problem. But, when a student is stuck, it's usually because he's stumbled on a particular difficulty. In order to identify this difficulty, we can compare the student's effective method to the information, which, combined to the ILM, can be found in the inferential graphs. So, by identifying the steps the student is unable to deduce, we can suppose that the difficulty lies in part in the knowledge he should have used. A second criterion consists in comparing the graphs of chosen sub-problems to keep only the one that sticks the best with the presumed difficulty. Consequently, we consider it to be advantageous to address the issue of the neighbourhood of the problems, complementing the other issues: the reasoning, the graphs and the ILM.

3.6 Learning Models and Diagnostic Model

In spite of any promise of a diagnostic of instrumented behaviour, the tutor agent claims to act on mathematical conceptions, which reveal themselves locally by the student-milieu interactions. In reference to the knowledge model of Balacheff and Margolinas (2005), the action of this agent relies on the problems, the solving operators, the representation systems, as well as the control structures, these being the four components of a conception. However, the development of mathematical competencies is a longterm project. If our system undoubtedly influences their evolution, it is not so much because of the tutor’s interventions, but mostly due to the choice of problems, which are the elements that globally create learning opportunities. This point of view leads us to consider that the learning model that ensues of the use of GGBT is bigger than its diagnostic model, this last one relying first on the student's conceptions. In other words, our system proposes to act on the development of mathematical competencies by offering a control on the acquisition of knowledge and an adaptation of the instructional model (learning opportunities) according to the student's instrumented behaviour.

4 Conclusion

Even if the backcloth of our ITS obviously is weaved around human learning and that we try to create a space of dialogue in the student-milieu exchanges, the discourse of the tutor agent overlaps with the discourse of the student without there being any real communication. Nevertheless, as a wink to Vigotsky, our research and development approach assures the existence of a zone of proximal development, in spite of the character essentially reactive of a tutor agent, which surfs graphs built a priori. If certain IT issues remain unanswered, like the ones evoked in the section *Problematic of GeogebraTUTOR*, we have to mention that the complexity of our approach forced us to defer the analysis, by the system, of the sentences written in natural language. For the moment, we work mostly on the other discursive aspects of the system and on the Iterative Learner's Model, as well as on the link between these and figural modelling. The integration of a computer algebra system, which depends on the current development of the GeoGebraCAS, and the organization of the cognitive messages are not in our priorities. However, the enrichment of the graphs by instrumented solving with real students is in progress.

Acknowledgements

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Figure Legend

Diagram 1. Situational map.

Illustration 2. Student's interface.

Diagram 3. System's structure.

Diagram 4. GGBT's rendering with regards to the steps of the student.

Figure 6. Inferential graph AND/OR.

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