Interactive Geometry inside MathDox

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Abstract

In this paper we describe how we envision using interactive geometry inside MathDox pages. In particular, by some examples we discuss how users and mathematical services (offered by various mathematical software packages) can interact with the geometric objects available. This not only includes manipulation of the geometric objects by users but also as a result of computations by computer algebra systems. Furthermore we discuss how manipulation of geometric objects can be used as input for queries to computational services to produce new views on a document.

Our approach is based on the OpenMath encoding of geometric objects and the InterGeo file format. The communication between Dynamic Geometry Software (DGS) and a MathDox page is realized by the means of an OpenMath phrasebook implemented in JavaScript.

Key words

Interactive mathematical documents, OpenMath, Phrasebook, interactive geometry

1 Introduction

New technologies such as JavaScript and Java and standards like XML, MathML and OpenMath have made it possible to bring mathematics to the Internet. What started with all kinds of interactive Java applets has resolved into highly interactive pages, on which the user can play and interact with mathematical formulas, graphs and other objects. The various web versions of Dynamic Geometry Systems (DGS) provide prominent examples of such interactive mathematical content. But also complete mathematical documents can nowadays be delivered as interactive web pages on which a user can profit from, for example, the computational power of Computer Algebra Systems (CAS).

Indeed, new formats like OmDoc and MathDox make it possible to create interactive web pages on mathematics. Both these formats are based on OpenMath, a semantic encoding of mathematical objects, and offer communication with various OpenMath aware systems. Within the MathDox system this communication is realized with the help of so-called OpenMath phrasebooks. These phrasebooks are thus able to provide services to an OpenMath aware system by invoking another such system.

As a result of the InterGeo project various DGS can exchange constructions using the common InterGeo file format. This file format maps to a set of experimental OpenMath

CDs the InterGeo consortium is developing in parallel to the file format. As a result, the DGS may be considered to be OpenMath aware.

In this paper we propose to create, next to the common file format, also a common JavaScript interface to the various interactive geometry packages, in the form of a JavaScript-based OpenMath phrasebook. Such a common phrasebook will make communication with other OpenMath aware software, such as various CAS and proof checkers, possible within HTML pages on the world wide web and will also extend the accessibility of web pages involving interactive geometry.

We have implemented a first experimental version of such a phrasebook in MathDox, a system developed at the Eindhoven University of Technology, see Cuypers et al. (2009). Below we describe some examples of how to use the interaction between a DGS and other OpenMath services available inside MathDox.

2 OpenMath technologies

The OpenMath standard language (Buswell 2004) provides a semantically rich representation of mathematical information for electronic access and usage. OpenMath was originally designed to be a common language for various CAS. However, over the last decade, MathML and OpenMath have become the predominant markup languages for mathematics (Caprotti_and Carlisle 1999) on the world wide web. MathML2 (MathML 2000) and its successor MatML3 cleanly define the interrelations between MathML and OpenMath. In fact, Content MathML3 and OpenMath are more or less isomorphic and can easily be exchanged.

The semantic information of OpenMath objects can be found in OpenMath Content Dictionaries (CDs). These OpenMath CDs collect and provide definitions of mathematical notions for usage within OpenMath applications. The official repository for these CDs can be found at www.openmath.org.

Both OMDoc (Kohlhase 2000) and MathDox (Cuypers et al. 2008) are XML formats for mathematical documents building on OpenMath. They both have mechanisms to present mathematical concepts in certain ways, for instance in hierarchies made of definitions, theories and theorems.

Both OMDoc and MathDox not only capture the semantics of mathematics, but also provide the user with the possibility to interact with mathematical services available through an OpenMath server, interfacing to a combination of one or more mathematical back engines. The server handles requests invoking computational aspects of a mathematical nature via OpenMath phrasebooks, programs interfacing the back engines to the client via OpenMath. See for example Caprotti et al. (2000, 2002, 2003).

A phrasebook recognizes a list of OpenMath CDs, and, within this OpenMath setting, it is able to perform several tasks, like an API. These tasks are specified in the *control information.* Examples of such tasks are EVAL, SIMPLIFY, PROVE, SOLVE, PRINT. This control information provides the interpretation of what is going to be done to the OpenMath objects by the back engines. The phrasebook translates the OpenMath and controle information into a well-defined task to be carried out by the back engine. It also specifies how the actual *communication* between the software package and the OpenMath computer environment is achieved, and finally delivers the result of the task carried out (in, for example OpenMath format) to its user.

Phrasebooks providing an interface to and from OpenMath are available for various mathematical software packages, e.g. Axiom, GAP, JACAS, Mathematica, Maple, Lego and COQ.

3 An OpenMath phrasebook for InterGeo

One of the main goals and achievements of the InterGeo project is a common file format for various of the participating DGS, including GeoNext, Geogebra, Cinderella JSXGraph and Cabri, see Abanades et al. (2009). This makes it possible that constructions made inside one of these packages can be saved in a file that can be understood by any of the other packages. There is also work on a common API.

The mathematical objects in the InterGeo file format also map to a set of OpenMath symbols defined in a set of (still experimental) CDs. As such the DGS involved in InterGeo do support OpenMath. To realize more interaction and communication between the various geometry packages and other OpenMath aware software, we propose to develop a full fledged phrasebook by adding *control information*.

As almost all of the DGS have a version which can be used inside a web browser, we want to implement this phrasebook inside a JavaScript program. Moreover, as most of the DGS already allow for JavaScript-based communication with a browser, this is just a natural extension of the common file format.

With the help of this phrasebook it should be possible to interactively add or delete objects from a geometric configuration, adjust objects or get information about them, if a DGS will allow it. In this way we can enrich a mathematical document on the internet with not only the beautiful and dynamic graphics of the geometry software, but also enhance it with the computational power of a CAS or capabilities of a proof-checker, or a combination of them.

Another advantage of this approach is that web pages in which the JavaScript phrasebook is used can be viewed using any of the supported DGS. A user that is used to a particular package, e.g. Geogebra or Cabri, can view the web page with his favorite Java applet displaying the geometric constructions. However, when he is on the road and only has an IPhone or IPad to his disposal, which does not support Java, he can view the same page using a package like JSXGraph.

4 Some examples

We now discuss some examples of how the interaction of a DGS and other OpenMath services available inside MathDox can enrich a mathematical document.

In our first example (displayed in the picture below) we have a MathDox page in which we encounter a configuration consisting of a straight line L defined by the points A and B and a parabola P. Moving the points A and B will of course also change the line L and hence the intersection points of the line and the parabola. To move the points, the user can use the dynamic geometry software, in this particular case JSXGraph, but he can also insert the new coordinates for the points A and B in the formula editor displayed in the picture. To determine the coordinates of the intersection points of the line L with the parabola P, one can of course rely on the computational power of JSXGraph. This, however, will result in approximations of the coordinates. If instead of JSXGraph, we use one of our OpenMath services, for example Mathematica or Maple, to do the computation, this will result in exact coordinates for the intersection points.

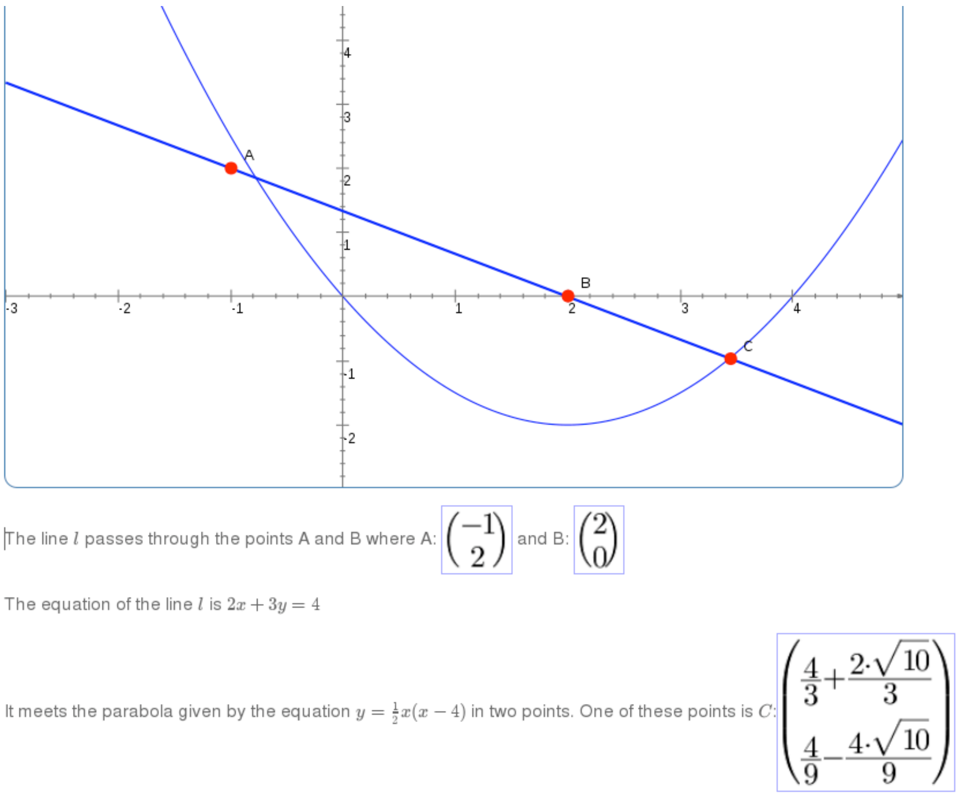

Figure 1. Exact computation of an intersection point.

A second example is the following. Consider the quadratic form defined by the symmetric 2x2-matrix M . The set of all points (x,y) with $(x,y)M(x,y)^T=1$ forms a (possibly empty) conic in the plane. The eigenvectors and eigenvalues of the matrix M determine the axes and shape of this conic. They can easily be computed by a computer algebra system and then displayed by dynamic geometry software. The user can play with the set up by changing the matrix M, or by manipulation of the conic (or its axes) within the dynamic geometry software. In this example we can see the interplay between geometry and (linear) algebra.

Our third and final example comes from calculus. The DGS can graph functions and also display areas between two graphs. The area can also be approximated. But with the help of a CAS one can compute the appropriate antiderivatives to obtain an exact formula for the area between two graphs (under the assumption that the functions used are from a well behaved class).

Clearly we can use the above examples both in theory pages as well as in exercises.

Of course there are many more applications one can think of. Indeed, connections of DGS with proof checkers and CAS can lead to automated proofs (Roozemond 2004) but also to a new class of automatically graded exercises on geometry.

5 Conclusions

As a result of the InterGeo project various DGS can exchange constructions using the common InterGeo file format. This file format maps to a set of experimental OpenMath CDs. As a result, the DGS may be considered to be OpenMath aware.

In this paper we have coined the idea to also provide a JavaScript based OpenMath phrasebook for these software systems, to enable a uniform way of communication between the software and a web page. In this way a mathematical web page will not only profit from the dynamic and beautiful graphic possibilities of the geometry software, it will also be able to combine them with other OpenMath aware mathematical software, like a Computer Algebra System or proof checker that can communicate with the web page.

Another advantage of this approach is that users are not bound to a specific DGS when visiting a web page. They may pick their favorite one, or the one supported by their browser.

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