Improving Undergraduate Students' Attitudes towards Geometric Proof through a Daily-Life Story using Dynamic Geometry Software

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Abstract

This work is a part of a larger study aimed at investigating the effectiveness of a suggested approach, which presents geometric problems through a daily-life story using dynamic geometry software for both school pupils and undergraduate students. The present paper aimed in particular to enable undergraduate students to feel the importance of geometry in daily life, to share in the process of formulating geometric statements and conjectures, to experience the geometric proof more than validating the correctness of geometric statements, and to start with a real-life situation going through seven steps to geometric proof. The content of the suggested approach was organized so that every activity is a prerequisite for entering the next one, either in the structure of geometric concepts or in the geometric-story context. Twelve undergraduate students from the Faculty of Education in Suez Canal University, Egypt participated in the study experiments and responded to *three Likert-type questionnaires*, which were prepared by the researcher with the purpose of assessing students' attitudes towards geometry and geometric proof, towards using computers in mathematics learning, and towards the suggested approach. The Wilcoxon rank test was used in the analysis of the ranked data collected from the three questionnaires as an alternative to the paired t-test. Results showed that there is no significant difference in general between pre- and post-administering of attitudes towards geometry and geometric proof. Contrarily, the analysis of single responses to questionnaire items showed significant changes in students' beliefs about geometry, geometric proof and towards using the suggested approach.

Keywords

Attitudes towards geometry and geometry proof, real-life geometry, dynamic geometry software, prospective-teacher education.

1 Introduction

The National Council of Teachers of Mathematics stated that "Geometry is a natural place for the development of students' reasoning and justification skills, where students should understand that a part of the beauty of mathematics is that when interesting things happen it is usually for a good reason" (NCTM, 2000, p. 42). In the United States, proof has been an important goal of the geometry curricula for more than a century (Senk, 1989, p.309), as well as in Egypt it plays a key role in helping students to construct mathematical structures (Refaat, 2001, p.63). The German curricula accepted that rigorous proofs should be avoided. Rather, it recommends that students should try ideas out, observe, guess and make reasonable justification (Kwak, 2005, p.14, p.37).

Unfortunately, despite reasoning being defined as several interrelated processes categorized as sense-making, conjecturing, convincing, reflecting, generalizing and justifying (Bjuland, 2002), the current situation in Egypt shows that asking students to write a formal proof is viewed as the main, even the first task of geometry class, which aims at verifying the correctness of one theorem followed by posing further exercises and solving them.

According to the professional experience of the present researcher, in the traditional geometry class, the teacher's role is just to introduce geometric concepts and theorems on the board and in the front of the classroom without any contribution from students in formulating this knowledge. That does not neither appreciates nor considers their minds. On the other hand, some of students are wondering that the geometric theorems are included in the textbooks and were already known as a work of famous geometers! So, it is obviously true and as a student "I am convinced with their trueness". Consequently, the false conception in the classroom regarding the function of geometric proof just to convince of rightness of theorems could be the reason behind a question arises in students' minds: "Why do we need to prove?"

In addition, the present researcher observed through interviews and discussions positive attitudes of students towards courses such as languages, biology, geology …… etc. They also describe these courses as interesting courses which tangible and needed in real-world. Contradictorily, they are wondering: "why do we need to learn geometry? When will we need to use Pythagoras or Thales theorems? This shows that they did not go through a learning experience that enables them to realize the importance of geometry in their real-life.

Hanna (1995) and Hersh (1993) described that the proof has two roles: one to convince that deals with *"what is true"* and another to explain that deals with *"Why is true?"* In view of that, Hersh (1993, p.396) differentiated between the role of proof in geometry class, and its role in research. While "*….In the classroom, convincing is no problem, students are easily convinced.*" and argued that the effect of proof learning experiences on students is often more emotional than intellectual and stated that *"if the instructor gives no better reason for proof than, that's math! How will the students find out why or wherefore, except: that's math!"*

Based on the findings of related studies, there are many indicators that students in geometry classes have neither the experience to see the importance of geometry in daily life nor to see the reasons to do geometric proof (Chazan, 1993& Almeida, 2000& Refaat, 2001& Nordström, 2003& Gfeller, 2005). The findings of Almeida's (2000) survey about the Swedish students' attitudes towards proof indicated that even with the general positive attitude toward geometric proof, the most negative students' response was *"I can't see the point of doing proofs: all the results in the course have already been proved beyond doubt by famous mathematicians"*.

Accordingly, what should now be considered are the learners' beliefs about geometry and geometric proof when the teacher asks them to prove that one theorem or geometric statement is true, and to write a formal proof. Numerous of related studies aimed to change students' negative attitudes. Actually, these studies accepted a "one dimension" definition of attitudes which related to as just students' emotions and feelings toward learning subjects and situations; see for example the study of Fan, L., Quek, K. S., Zhu, Y., Yeo, S. M., Lionel, P. & Lee, P. Y. (2005) that aimed at assessing Singapore students' attitudes toward mathematics and mathematics learning, the researchers gives ten examples from the whole study questionnaire items, which totally focused on the affective domain while they used expressions such as: I enjoy, I am not afraid, have confidence, I like, and I am sure. These expressions showed that the researchers accepted the simple definition of attitudes towards mathematics as just students' emotions.

In one attempt to investigate more components of attitudes and the relation between them and achievement in Euclidean Geometry, Mogari (1999) examined four components of attitudes namely: enjoyment, motivation, perception of the importance of geometry and freedom from fear of geometry. The results indicate that there were very weak relationships between achievement and each of the four variables. According to the multicomponent definitions of attitudes, the present researcher consider the four variables in Mogarti's study as only two of attitude's components while enjoyment, motivation and freedom from fear of geometry could be categorized as emotions, and perception of the importance of geometry as merely one element in the beliefs about Euclidean geometry.

Pickens (2005, p.44) went beyond this simple view of attitudes, and clarified that attitudes might help us to define how students can see the different situations, how they behave in these situations and reflect their feelings, thoughts, and actions. So, attitudes give a view about the structure of the internal beliefs, thoughts, and how these two components affect students' behavior in the different situations. Di Martino and Zan (2003) agree with this "multicomponent" definition, which included both of the affective, the cognitive components and the behavioral tendency. Zan and Di Martino (2007) came up with further explanations and articulated that in most questionnaires of attitudes, an answer can be characterized as a positive answer and that might refer to different meanings. These meanings varied depending on what the word positive refers to: emotions, beliefs or behaviors. So, when the word positive refers to emotions such as feeling anxious in doing mathematics, it is seen as negative. In contrast of that, pleasure will be seen as positive. When the word positive refers to beliefs, it is generally deals with shared meanings and ideas. While the successes in learning contexts that usually identified with high achievement reflects the meaning of the word positive when it refers to positive behavior.

Gómez-Chacón and Haines (2008) have been clarified the meaning of positive or negative attitude according to the relationship between the previous components, so the attitude of one student who likes mathematics defined as "negative", if this positive "emotions" are associated with false beliefs about mathematics, for example as a set of rules to be memorized. The present researcher wants to emphasize on this indication, while positive emotions and positive behaviors "success and high achievement" could be an outcome for learning and teaching methods that emphasize rote learning, which limit the rule of mathematics in just memorize and recall it in the final exam. In this case, the students might expected to have positive emotions, positive behavior "participating in the classroom to memorize the information, and high achievement in the final examinations which required just to recall such concept definitions and theorems and even proofs" but the overall outcome considered to be as a negative attitudes because the both of positive emotions and behavior depended on a false beliefs about the function of mathematics.

That shows the interrelated relationships between these components, so that beliefs affect emotions and both beliefs and emotions affects behavior (Pickens, 2005, p.44). Goldin, Rösken and Törner (2009) indicated that there is a need for research on the roles that beliefs might play in order to open up possibilities of deeper mathematical understandings and evolve students' attitudes and achievement. Although, this review could provide an answer to the question: Why do geometry and geometric proof often have a similar tone of negative emotions and false beliefs among learners and even teachers themselves?

While there was agreement around the world on the importance of geometry and geometric proof regardless of the diversity of educational systems, another question remains: How could the new trends of instruction technology play a part in changing these attitudes? Thus, the present study is an attempt to improve students' emotions and change their false beliefs about geometry and geometric proof. It also aims to benefit from the facilities of dynamic geometry software, in order to activate the role of constructivism learning theory in engaging students actively in real-life geometry learning situations. These dimensions (Emotions, Beliefs and Students' participations) show the components of the attitude (Emotions, Beliefs and Behaviours) according to the multicomponent definition (see Di Martino and Zan, 2003, p. 453).

Why real-life situations?

The National Council of Teachers of Mathematics reported that everyone can use geometric ideas to represent and solve problems in the real-world and also that studying measurement is important because of its practicality in so many aspects of everyday life. Furthermore, the report lays emphasis on the importance of recognizing, applying and making connections between mathematics and real-world contexts (NCTM, 2009). Sun and Williams (2003) referred to the importance of constructivist learning in acquiring knowledge that required learner-cantered, goal-directed, and real-life problems in order to find meaningful solutions. Kanuka and Anderson (1999) considered the importance of the learners' prior knowledge in applying their understandings in learning activities that have real-world relevance for them. This obviously showed the role of real-life-centered learning in order to stimulate students to actively participate in geometry class. Therefore, the present study sought to engage students in real-life geometry situations, which could change beliefs about geometry from it just consisting of learning tasks inside the classroom to feeling its importance in real life, and to feel that doing geometry and geometric proof are not just professional tasks limited to mathematicians.

Yet, it seems that engaging students in real-life situations is not enough to meet the needs for geometrical understanding& as Kemeny (2006) explained, establishing a connection between instruction and real-world requires hands-on activities. Consequently, designing instructional environments that evolve students' experiences from just recognizing real-life geometry situations to the level of doing geometry and proving becomes the challenge. Many studies have been conducted to investigate the facilities and the effectiveness of using several learning environments to overcome the inadequacy in mathematics teaching and learning especially in geometry (e.g., Noss, 1988& Refaat, 2001). However, studies are scarce that engage students in real-life geometry situations to overcome difficulties in geometry and geometric-proof learning as well as improving the attitudes towards these subjects in mathematics classrooms (e.g., Stillman, 2006& Pierce & Stacey, 2006& Duatepe-Paksu, Ubuz, 2009). Obviously, developing a geometry learning environment that enables students to participate in hands-on activities is needed in the context of the present study.

Why dynamic geometry software?

Previous studies have indicated the impacts of using dynamic geometry on problem posing and solving (Christou et al., 2005), understanding geometric concepts and theorems (Almeqdadi, 2000), discovering and conjecturing (Aarnes, and Knudtzon, 2003& Furinghetti and Paola 2008& Habre, 2009), achievement and attitude towards mathematics (Phonguttha et al., 2009) and on developing reasoning and proof abilities (Jiang, 2002). Jones (2001) summarized the didactical side of these facilities which could help students to participate in an interactive learning environment that might reduce the gap between geometrical construction and deduction, gain a more meaningful idea of proof and proving.

Kortenkamp (1999) clarified the facilities of dynamic geometry software, which opened up new opportunities for computers use in the field of mathematics teaching and learning and especially in regard to geometry, and stated that using the mouse makes it possible to draw geometric elements such as lines, circles, and intersections. In addition to loci and traces of these objects that could be displayed on the computer screen. Furthermore, the facilities of dynamic geometry software enable the user to construct accurate geometric constructions, while the possibility of dragging elements of geometric configurations is the fundamental advantage that magnifies the ability to explore the behavior of a construction by moving its elements in comparison to the traditional ruler and compass. Almeqdadi (2000) concluded that the use of dynamic geometry software can create an effective learning environment that provides students with a good simulation, which is much closed to the real-life situations. Furinghetti (2008) explained that the possibilities to motivate students to prove throughout dynamic geometry activities is different from those in the traditional methods, while the motivations provided in such interactive activities are similar to those that one mathematicians have.

For the present study dynamic geometry software was chosen as a means to present geometric concepts and theorems in real-life situations, since it may provide the students with an appropriate learning environment, in which they can feel the possibility of doing geometry, formulating geometric statements by themselves and realizing the relevancy of geometry in real life.

In constructivism terms, Hay and Barab (2009) assumed that "… by the degree of active learner engagement as well as the assumption that learners have the ability to create meaning, understanding, and knowledge. Students are not passive receptacles of the knowledge that teacher impart". Accordingly, when students play an active role in rediscovering the geometric concepts and theorems by themselves, they will not only be able to realize the verification function of proof, but also be motivated to explain why their own conclusions are true. De Villiers (2003) shed light on this idea and clarified the usefulness of dynamic geometry software, when he stated that is *"enables students to experiment through unlimited variation construction and makes conjectures. So that the next task is to investigate whys their conjectures are true& while the dynamic geometric construction is simply sufficient for convincing with the trueness of conjectures"*.

Why the combination?

The National Council of Teachers of Mathematics standards emphasized the use of concrete models using dynamic geometry software to explore geometric ideas and realize their usability in real-world contexts (NCTM, 2009). Leading from that, this study included the use of dynamic geometry software in presenting geometric concepts and theorems in real-life situations. This combination may provide students with an appropriate geometric learning environment in which they would actively participate in formulating geometric statements and conjectures by themselves, recognizing the idea of the proof. Moreover, the facilities of dynamic geometry software are able to model the real-life geometric situations, which bring the geometric concepts and theorems to life and bring the real-life geometric situations into the classroom.

Such a learning context might enable students to use their previous experiences and knowledge to discover and share geometric ideas, which activate the principles of learning theories such as social- and cognitive constructivism. In terms of social constructivism, the present researcher expects that using such an approach might engage students in a social learning context, which is essential to improve the language required to express their thinking and writing down geometric explanations. The NCTM's guiding principles indicated the importance of using language to express ideas precisely and organizing mathematical thinking through communication, stated that *"When students communicate the results of their thinking with others, they learn to be clear and convincing in their verbal and written explanations"* (NCTM, 2009).

2 The Suggested Approach

The suggested content organized in a series of real-life situations that formed a geometric story (see Figure 1) followed by abstracted geometric theorems depended on the knowledge acquired from the real-life geometry activities. All of the presented activities were based on dynamic geometry facilities. This approach aimed at helping students to: feel the importance of geometry in real-life situations, formulate geometric statements and conjectures by themselves, modify their negative emotions and false beliefs about geometry and geometric proof, overcome one of the most difficult steps in geometric proof – which is to get the idea of the proof, e.g., how to start proving –, recognize visual explanations of the statements and write them down in their own language, unify and adopt the language of mathematical explanations during geometry class discussions and scaffold their geometric proof writing. Seven geometric activities formed the whole geometric story content [iii] followed by six geometric theorems activities that were developed during the period of a PhD scholarship in Germany, funded by the Ministry of Higher Education in Egypt (from 2007 to 2009) & two real-life geometric situations were adapted from De Villiers (2006) and Herrmann (2005) to fit the study context. These situations are "the water tank problem" and the "mirror problem". Due to the time limit of the study experiments, only four activities were administered. At the beginning of the experiment, twenty one undergraduate students from the mathematics section in the Faculty of Education in Suez Canal University in Egypt were introduced to the purposes of the study and responded to the pre-test, but only twelve students were available for the post-test [iv]. In the following is a summary of the geometric story followed by an example that shows the learning phases:

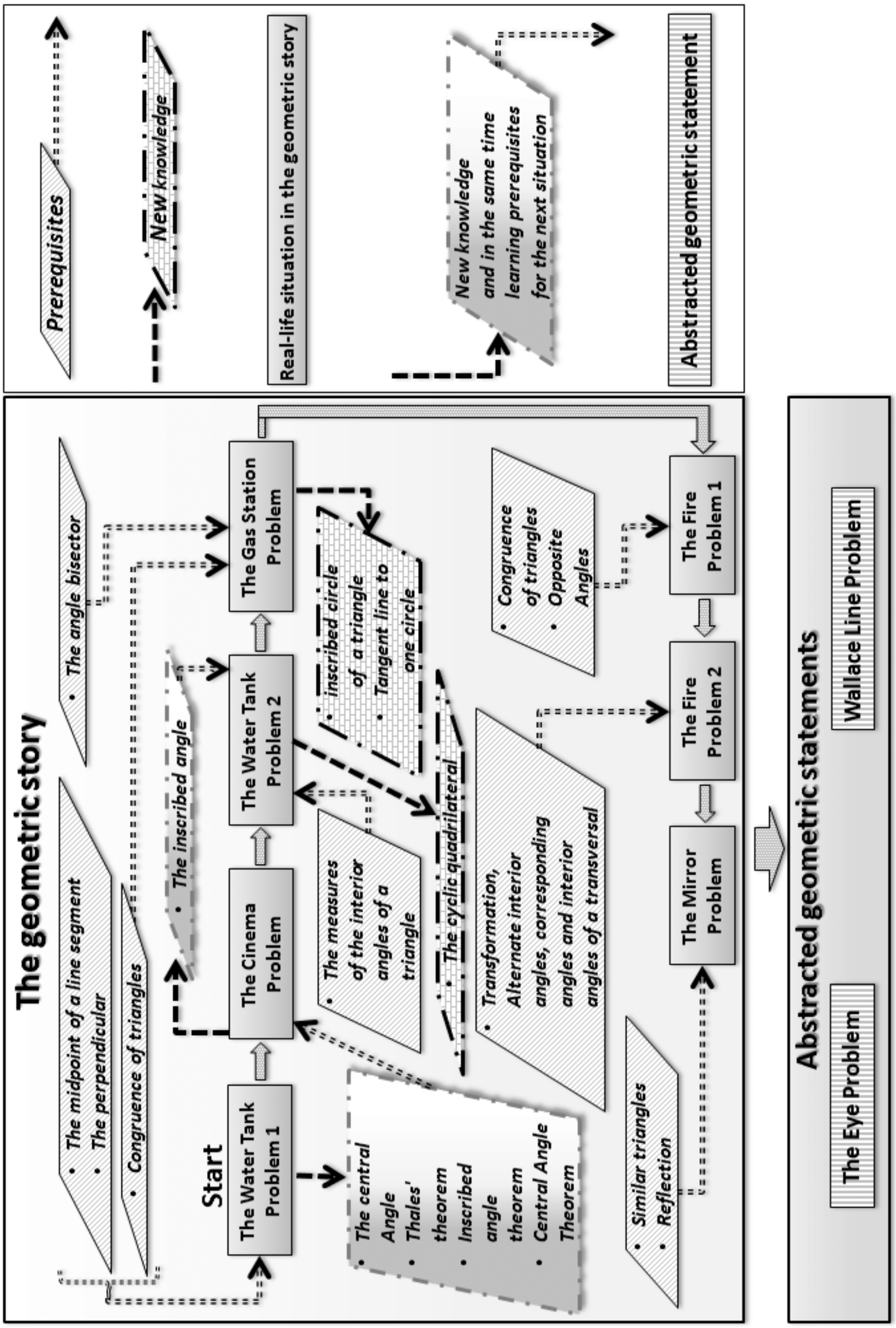


Figure 1: The schematic structure of the suggested content.

3 The geometric story (a new city)

Situation 1, the Water Tank Problem 1:

In a place in the desert, there is a new triangular city consisting of three housing areas. The administration of the city decided to build a water tank to serve a maximum of five housing areas. Beyond the housing areas, no more houses are allowed to be built. The water tank should be at the same distance from all three areas.

Hence, the first problem, the administration faced, was how to determine the place of the water tank that is exactly between all three housing areas?

Situation 2, the Water Tank Problem 2:

The administration would like to extend the city with a new housing area, which has the same distance from the water tank as the other three areas. What shape do the four housing areas form in order to have the same distance from the water tank?

Situation 3, the Cinema Problem:

The administration then decides to build a cinema. Think about a geometrical explanation that gives a reason for the administration's choice of an amphitheater's shape for the cinema.

Situation 4, the Gas Station Problem:

The administration would like to build a gas station between the three main roads, joining the three areas. Where is the optimal point that has the same and shortest distance from the three roads?

Situation 5, the Fire Problem 1:

After it had been built, a fire broke out in the gas station: there are two possible options for extinguishing it. In the first option, the firefighters and the burning gas station are on the same side of the river.

Situation 6, the Fire Problem 2:

In the second option, the firefighters are on the opposite side of the river. Considering that the firefighters need the water from the river in order to extinguish the fire, which point on the river is optimal for shortening the distance between the fire and the firefighters?

Situation 7, the Mirror Problem:

A man, unaware of the previous events in the city, is building his house in one of these housing areas. He is interested in hanging a mirror of a certain size in the entrance, so that he can see himself completely in it when entering the house. What do the dimensions of the mirror should be, to satisfy the man's needs?

4 Learning phases

The main goal of the suggested approach is to make a smooth transition from approaching every real-life situation in the geometric story to the level of doing geometry and geometric proof. The following shows the learning phases using the first situation as an example:

Phase 1: Approaching a real-life situation

In this phase, students were able to discuss and realize a scene of the whole geometric story that required doing geometry, in order to set up the water tank exactly equidistant from all the three housing areas.

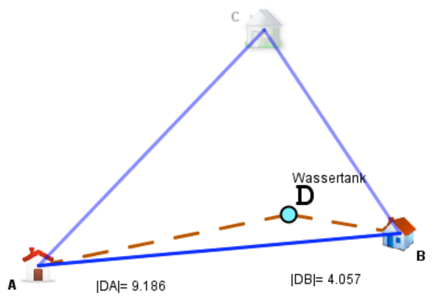
Phase 2: Experimenting and conjecturing

After having discussed the situation, students moved on to the experimentation phase, which could change students' attitudes toward geometry as not being just a professional task carried out only by geometers. Here each student paired up with a partner and shared their conclusions. This phase was divided in the current example into three parts. In every part students were encouraged to drag point D "the water tank" in order to determine the possible locations for the point that makes D equidistant from the three housing areas. The instructions were produced for students as follows:

Experiment 1:

The water tank at point "D" should have the same distance to all three housing areas.

We begin with a simple task. First, you should find where the best point is that will place the tank in the middle between the two housing areas.

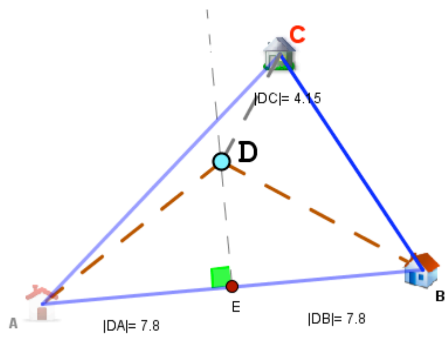


• Move the blue point so that the tank lies exactly in the midpoint between the two areas. Write down your observation in the given handout.
Note: The blue point is a movable point.

• Compare the results with your class partner and then with the whole class. **Write down the possible solutions with configurations in the given handout.**

Experiment 2:

There are many possible points that satisfy the condition that the tank lies equidistant from the two housing areas "A and B", these points are collinear. Which point in your opinion is the optimal point if we also include the third housing area "C" in our calculations?



• Change D, so that AD = BD = CD. What do you think? **Write down your conjecture.**

• Change the form of the triangle ABC and move point "**D**" again. Is your conjecture still correct?

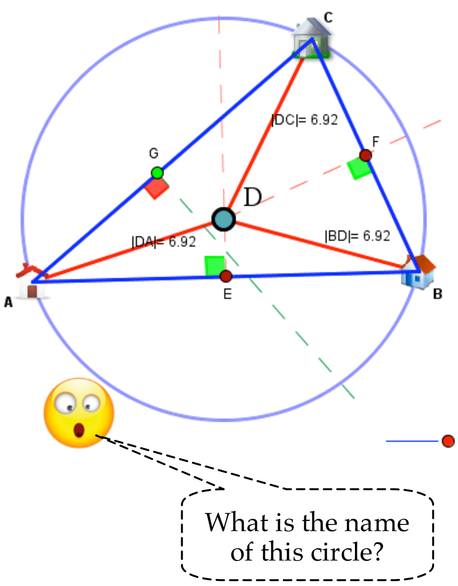
Write down your conclusions with their configurations in your handout.

Experiment 3:

Is your conjecture still correct so that the tank is equidistant from the two housing areas **A und C**?

Does **D** also lie on the perpendicular bisector of **AC**?

"The green point G is a movable point"



• Deform the triangle ABC; is your conjecture still correct?

• Investigate why the intersection point **"D"** of the three perpendicular bisectors is also a "Center".

Hint

Move the red point, which is in the bottom right corner of the figure to the right

What is the name of the circle?

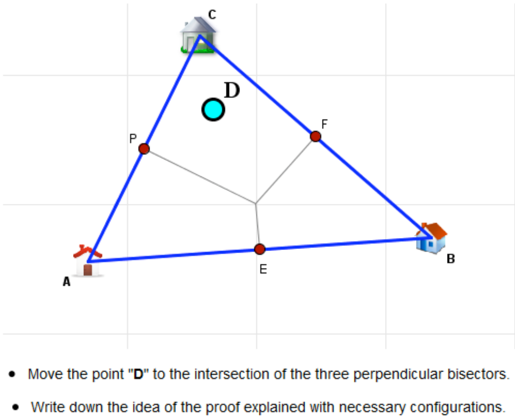
Note your conclusions with configurations.

Phase 3: Reformulating conclusions

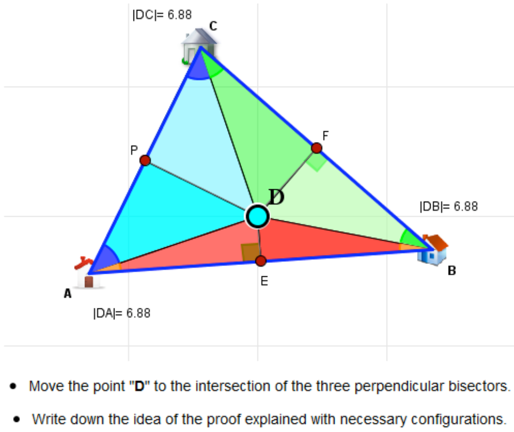
In this phase all of the students were invited to participate in a classroom discussion. This phase might eliminate students' negative emotions if their observations or conclusions are different to those of other class members, while the teacher's questions and comments also play a vital part in giving feedback and unifying the students' responses. This phase could also facilitate a smooth transfer from students using their own language to adopting the use of the appropriate mathematical language.

Phase 4: Getting a proof idea

In order to get a proof idea, dynamic geometry software with its dragging and measuring features, and the included programing language scripts could provide the appropriate learning context to magnify students' ability of visual recognition for the geometric constructions and learn the key ideas which will allow them to start proving.



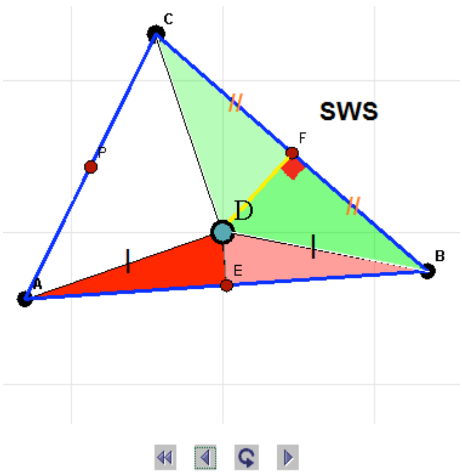
Before dragging



After dragging

Phase 5: Visual explanation

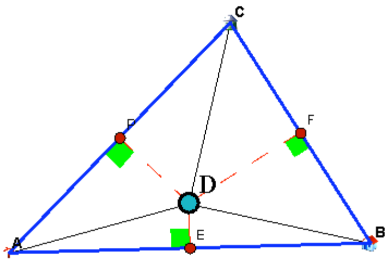
In this phase students were invited to explain what they had visually concluded from each step in sequenced illustrations and then to write it down in their own language.



- Note each step in the given handout.
- Try to explain each step.
- Write down the proof by combining these steps.

Phase 6: Scaffolding proof writing

Making use of JavaScript and image hover effect, this phase as shown below aimed at evolving students' ability from the use of their own language to justifying their explanation and scaffolding proof writing step by step. This was not a standalone self-learning phase as the teacher's questioning played an important part in guiding students from one conclusion to the next. Also, inviting students to rewrite their explanation using the appropriate mathematical language was a very important hint.



Given that:
$\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, and $\overline{DP} \perp \overline{DF}$.

R.T.P :
The point "**D**" is the center of the circumcircle of the Δ ABC.

Proof:
The points E, F and P are the midpoints of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively.

Considering Δ ADE.

- What is the congruent triangle to Δ ADE? Hint

$\overline{DE} \perp \overline{AB}$, and $|\overline{AE}| = |\overline{EB}|$ **SWS**

- Now, what can you conclude about $\overline{AD}$ and $\overline{BD}$? .... (1)

Similarly: Considering Δ BDF.

- What can you conclude about $\overline{CD}$ and $\overline{BD}$? .... (2)
- What can you further conclude from steps (1) and (2)? ..... (3)
- **From (1), (2) and (3), prove in the given handout.**

Click here for the complete proof.

Phase 7: Comparing with the complete proof

After having justified their explanations and rewrote them using a better mathematical language than in the 5th phase, students were invited to compare their proof with the complete proof provided in a PDF file to check any mistakes.

5 Method and Study Procedures

Three questionnaires of attitudes (towards geometry and geometric proof, learning using computers and towards the suggested approach) were prepared and used in a pre-intervention-post design. The questionnaire of attitudes toward geometry and geometry proof consisted of twenty Likert items and an open-response item. The questionnaire of attitudes towards learning using the computer consisted of fifteen Likert items and an open-response item, whereas the questionnaire of attitudes towards the suggested approach consisted of twenty Likert items and an open-response item. The questionnaires were developed during seminar discussions with staff members in Bayreuth University in Germany and mathematics teachers in Germany and Egypt, to state how far the questionnaires' items measure what they were intended to measure.

Every questionnaire's reliability was determined from a prior administration on students in Suez Canal University (n=14), Egypt. The researcher used Cronbach's coefficient to calculate the reliability of the questionnaires through a pilot study. In addition, the statistical item-total correlation analysis was used to determine the experimental validity.

The reliability coefficient was calculated using SPSS17. For the questionnaire of attitudes towards geometry and geometric proof a mediate reliability coefficient was calculated at 0.775& for the questionnaire of attitudes towards using the computers in learning a low reliability coefficient was calculated at 0.604. The reliability of the questionnaire of attitudes towards the suggested approach was 0.936, which was a high reliability coefficient. As a result, some items of each questionnaire were deleted or revised. [ii]

6 Study Hypotheses

To examine the effect of the suggested approach on students' attitudes towards geometry and geometric proof, two null hypotheses were formulated and tested:

1. There are significant differences between mean ranks of pre-and-post questionnaire of attitudes towards geometry and geometric proof.

2. There are significant differences between the mean rank of attitudes towards learning using the computer and the mean rank of attitudes towards using the suggested approach.

7 Results and Discussion

In order to accept or refuse the above hypotheses, the Wilcoxon rank test and the effect size coefficient were used in analysing of the ranked data collected through the three questionnaires, as an equivalent of the paired t-test in addition to the descriptive statistical analysis of the three questionnaire items. The statistical package for social sciences SPSS was used for inputting and processing the data collected from the questionnaires before and after intervention.

Towards Geometry and Geometric Proof

Wilcoxon Ranks Test

Ranks		N	Mean Rank	Sum of Ranks
After -Before	Negative Ranks	4[a]	2.50	10.00
	Positive Ranks	5[b]	7.00	35.00
	Ties	3[c]		
	Total	12		

a: After < Before, b: After > Before and c: After = Before

Test statistics [b]

	After- Before
Z	-1.482[a]
Sig.	
(2-tailed)	0.138

a: based on positive ranks and b: Wilcoxon Ranks Test

Based on the coefficients produced from the above tables, the results indicated that there are general differences in students' attitudes towards geometry and geometric proof, z = -1.482, but these differences were not significant because the significant coefficient Sig. = 0.138 higher than the confidence interval = 0.05, this result showed that individual item analysis is needed in order to investigate these differences. Since, significant differences in individual item's responses may be existing.

The students' average responses to item thirteen "Often, I wonder: "Why should I prove a statement, which is obviously true and has already been proved by famous geometers!" the average response ranged between "Agree" and "I do not know" in the pre-test, while it was "Disagree" in the post test. The average response for item fourteen "I think it is more important to use geometry statements to solve exercises more than to prove them" ranged between "Agree" and "I do not know" in the pre-test, while it was "Disagree" in the post test. For item twelve "Geometric proof gives me explanations about the trueness of geometric statements" was between "Strongly disagree" and "Disagree" in the pre-test while it was "Agree" in the post test. This shows the changing in students' conceptions about the importance and the functions of geometric proof. These results are consistent to some extent with Almeida's (2000) conclusions& however, despite general positive attitudes towards geometry Almeida's questionnaire results indicated that students have false beliefs about the function of geometry proof which are limited to the verification function. The results of the present study show that regardless of the non-significant differences in students' attitudes towards geometry and geometric proof, the previous three item responses indicate a change in students' beliefs, especially in regard to the function of geometric proof. This result shows the need for further investigation, and reflects the need for longer intervention in order to improve students' attitudes towards geometry and geometric proof.

Between using computer in learning and the suggested approach

Wilcoxon Ranks Test

Ranks		N	Mean Rank	Sum of Ranks
Suggested Approach-Computer	Negative Ranks	1[a]	1.00	1.00
	Positive Ranks	11[b]	7.00	77.00
	Ties	0[c]		
	Total	12		

a: After < Before, b: After > Before and c: After = Before

Test statistics [b]

	Suggested Approach - Computer
Z	-2.981[a]
Sig. (2-tailed)	0.003

a: based on positive ranks and b: Wilcoxon Ranks Test

The coefficients in the previous tables show a significant difference between students' attitudes towards using a computer in learning and towards the suggested approach which they intended, while the coefficient Sig. = 0.003 is significantly higher than the value of $\alpha = 0.05$ in most of the previous studies and even if compared with the value of $\alpha = 0.01$. This result invites the researcher to use the following equation in order to compute the effect size:

$r = Z/\sqrt{N}$, N is the number of students and Z is Z-score of Wilcoxon Ranks Test in SPSS output.

Thus $r = -2.981/\sqrt{12} = -2.981/3.46 = -0.86$

The coefficient "r" according to the table of critical values for the Wilcoxon test indicates that this difference is substantial and shows positive attitudes towards using the suggested approach which contains the real-life geometric situations and is based on dynamic geometry software.

This result could be a logical interpretation of responses from most of the students - before intervention - to the following item in the questionnaire about using computers in learning: "Have you had any experience with any mathematics learning software?" – Most of the answers were "no". Furthermore, students' responses to an open-ended question about using computers in learning reflected their negative attitudes towards using computers in general and showed that they had not any experience with them in this context before. However, there was a negative response to one item "Dynamic geometry activities allow me to learn in my own learning tempo" which for most students was between "I do not know" and "Disagree"& this might be a disadvantage for the pre-design and the linearity of the suggested approach.

Briefly, the analysis of students' responses as a whole does not indicate that the use of the suggested approach had significant differences in their attitudes towards geometry and geometric proof as a whole. The limited time of the study might explain this result, while the study experiments were completed during four sessions in around four weeks. Thus, students may not have had enough time to learn using the suggested content. On the other hand, the results indicated that the students' attitudes towards the suggested approach were significant and substantial. Unfortunately, there was a negative response to one item that investigated the suitability of the learning time and learning tempo. Accordingly, the present researcher has some recommendations and suggestions to develop the whole study and for future studies summarized in the following points:

1. Future studies conducted over a longer period of time may produce different results.
2. Allow students to participate in situation structuring or in constructing the dynamic geometry applets& the students who participated in the present study had no previous experience with using dynamic geometry or even computers in mathematics learning.
3. The present study can be replicated to investigate attitudes towards geometry and geometric proof in each level of geometric thinking.

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(i) Curriculum vitae available at: http://www.dynamicgeometry.net/cv/

(ii) The final versions of the questionnaires and reliability statistical analysis available at: http//:www.dynamicgeometry.net/que/

(iii) The whole story and one interactive example available at: http//:www.dynamicgeometry.net/exm/

(iv) Some videos of students' participations available at: http//:www.dynamicgeometry.net/vid/