

Using Automatic Theorem Proving to improve the User Interface of Geometry Software

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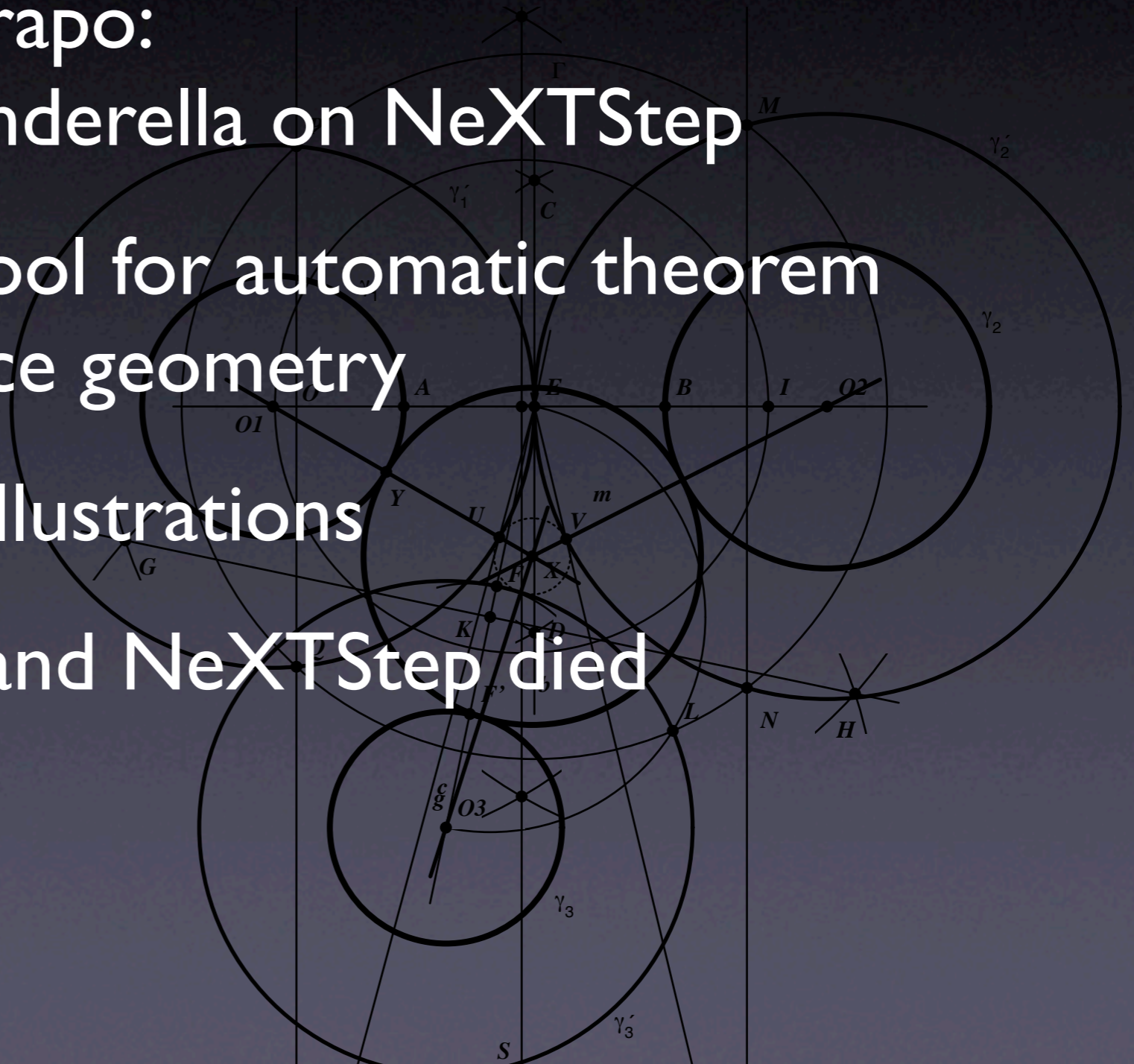
... travel back in time ...

- *Cinderella*: a user interface for geometry
- 1993-1997
a user interface for automatic theorem proving
- 1997-2003
Internet-based e-Learning geometry software
- 2003-...
both



1993-96: *Cinderella* NeXT

- Richter-Gebert/Crapo:
first version of *Cinderella* on NeXTStep
- used as an input tool for automatic theorem proving in incidence geometry
- used for drawing illustrations
- not distributable, and NeXTStep died



Cinderella 0: 1996-1997

- Java-based replacement for *Cinderella* (NeXT)
- used for giving demos
- 1996: submission for Multimedia-Award stimulated the extension to eLearning
- publishers asked for commercial version
 - Easy to deliver?



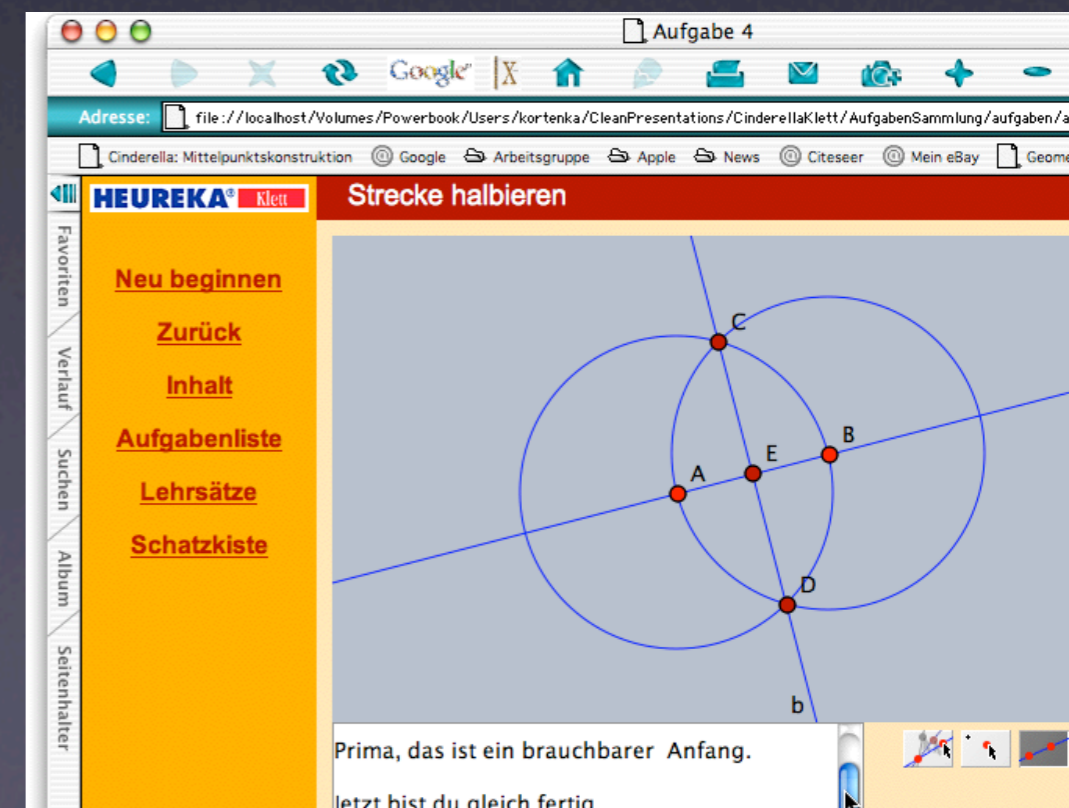
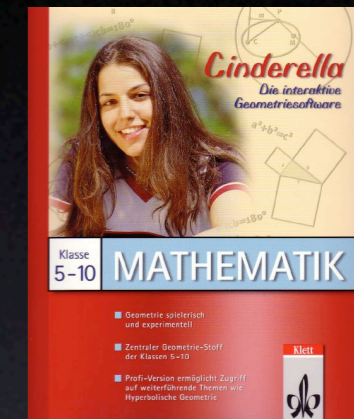
Cinderella 0: Problems

- identical elements can appear in the construction
- incidences are only found on request
- using the symbolic prover is awkward
- the rendering of loci may be slow and ugly
- exercises are custom-made



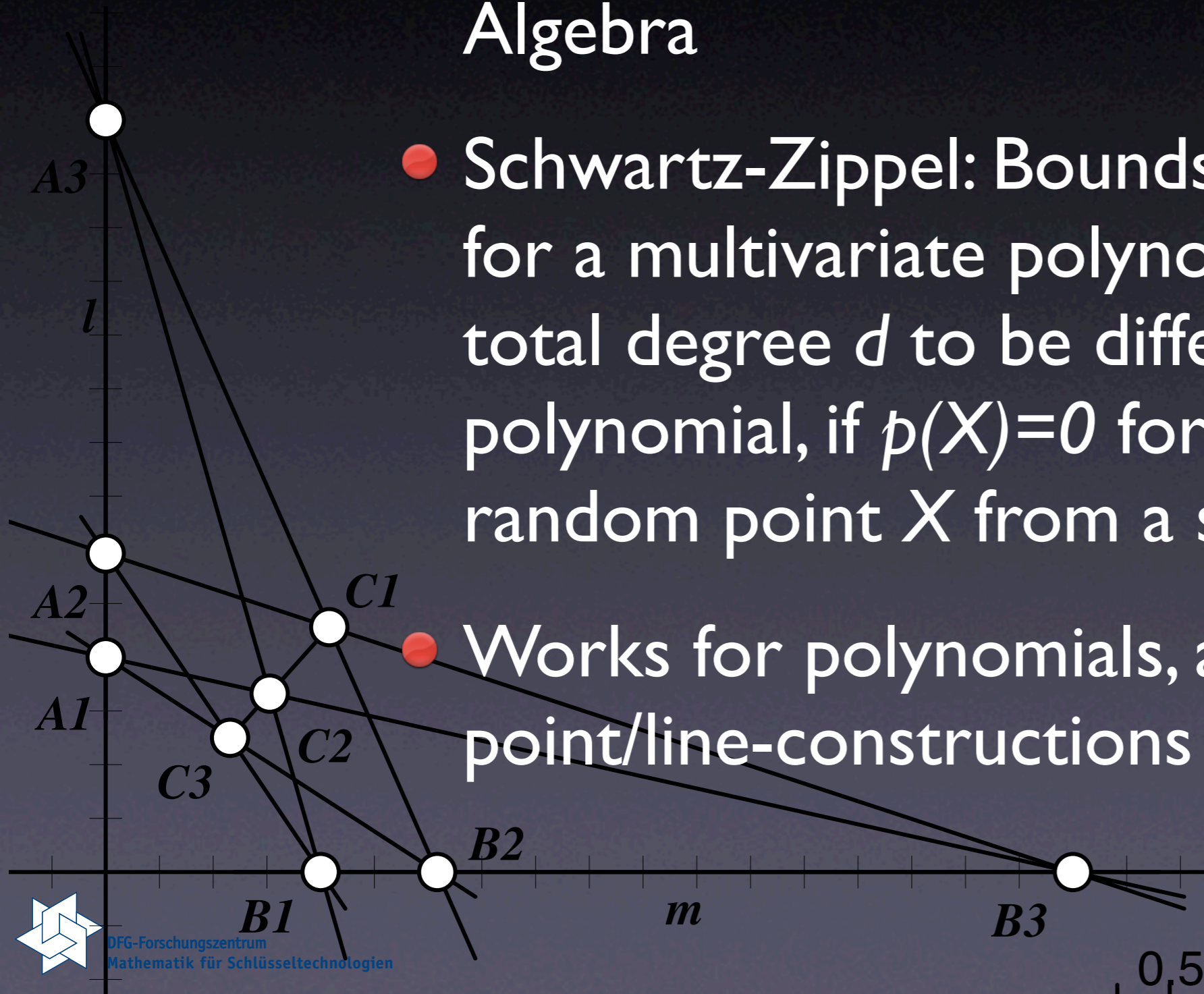
Cinderella I: 1998-2003

- Commercial Software
- User interface backed by automatic theorem proving
- new educational features based on theorem proving



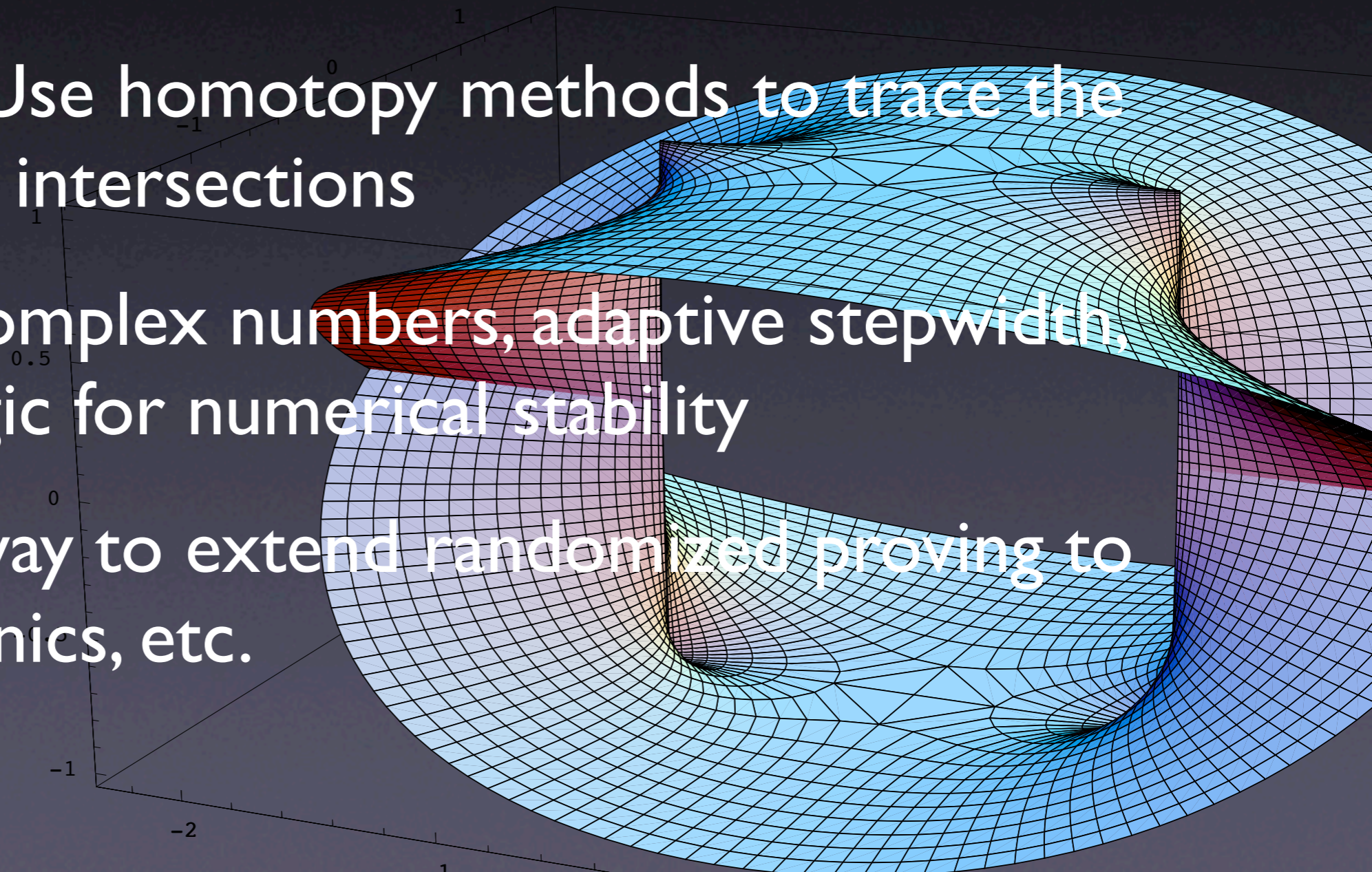
Randomized Proving

- Extension of Fundamental Theorem of Algebra
- Schwartz-Zippel: Bounds the probability for a multivariate polynomial p of maximal total degree d to be different to the zero polynomial, if $p(X)=0$ for a uniformly random point X from a set S .
- Works for polynomials, and thus also for point/line-constructions



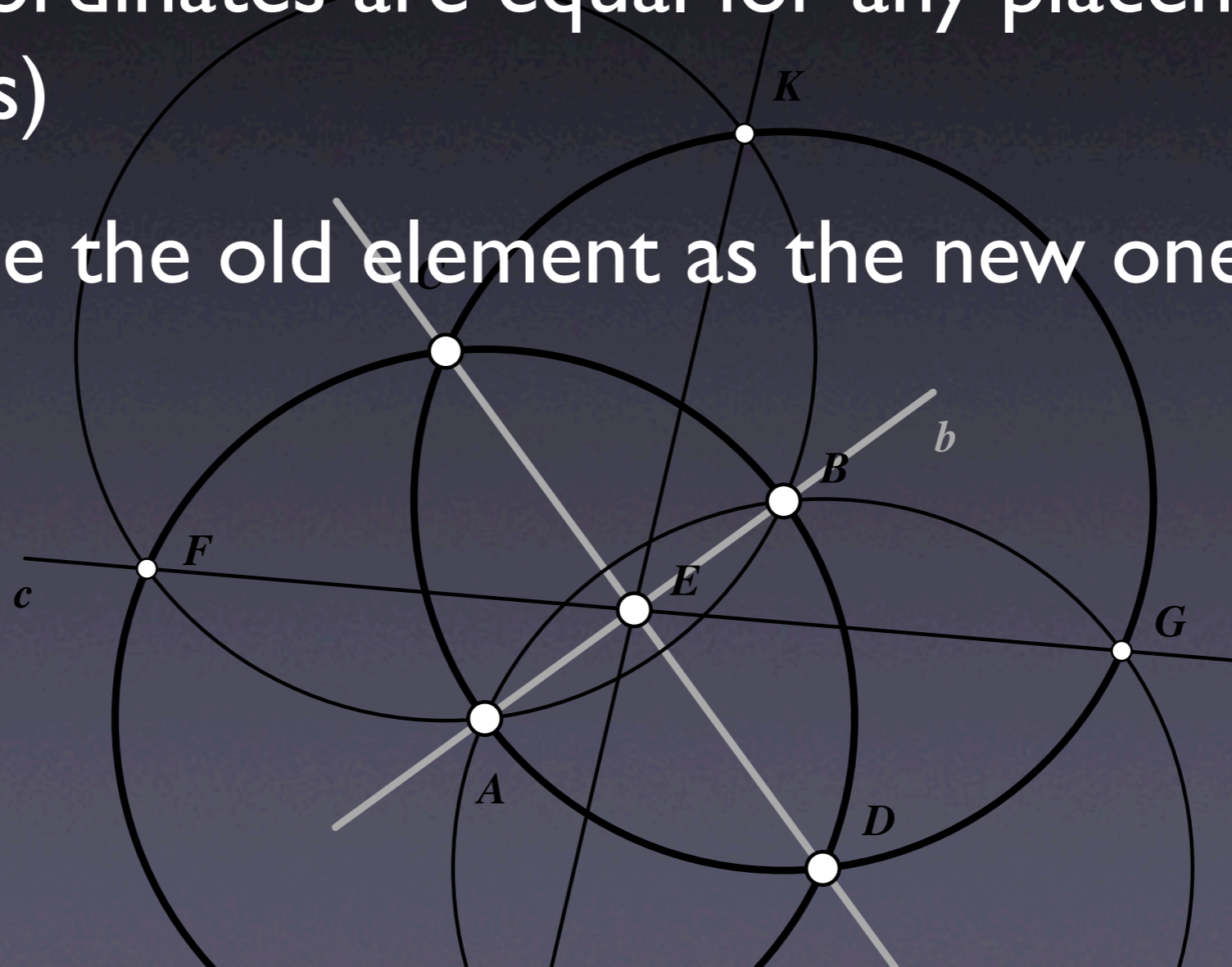
Continuity in DGS

- Problem: There is no straightforward way to select the right intersection from a set of multiple intersections of two, say, conics.
- Solution: Use homotopy methods to trace the path of all intersections
- Needs: Complex numbers, adaptive stepwidth, some magic for numerical stability
- Gives: A way to extend randomized proving to circles, conics, etc.



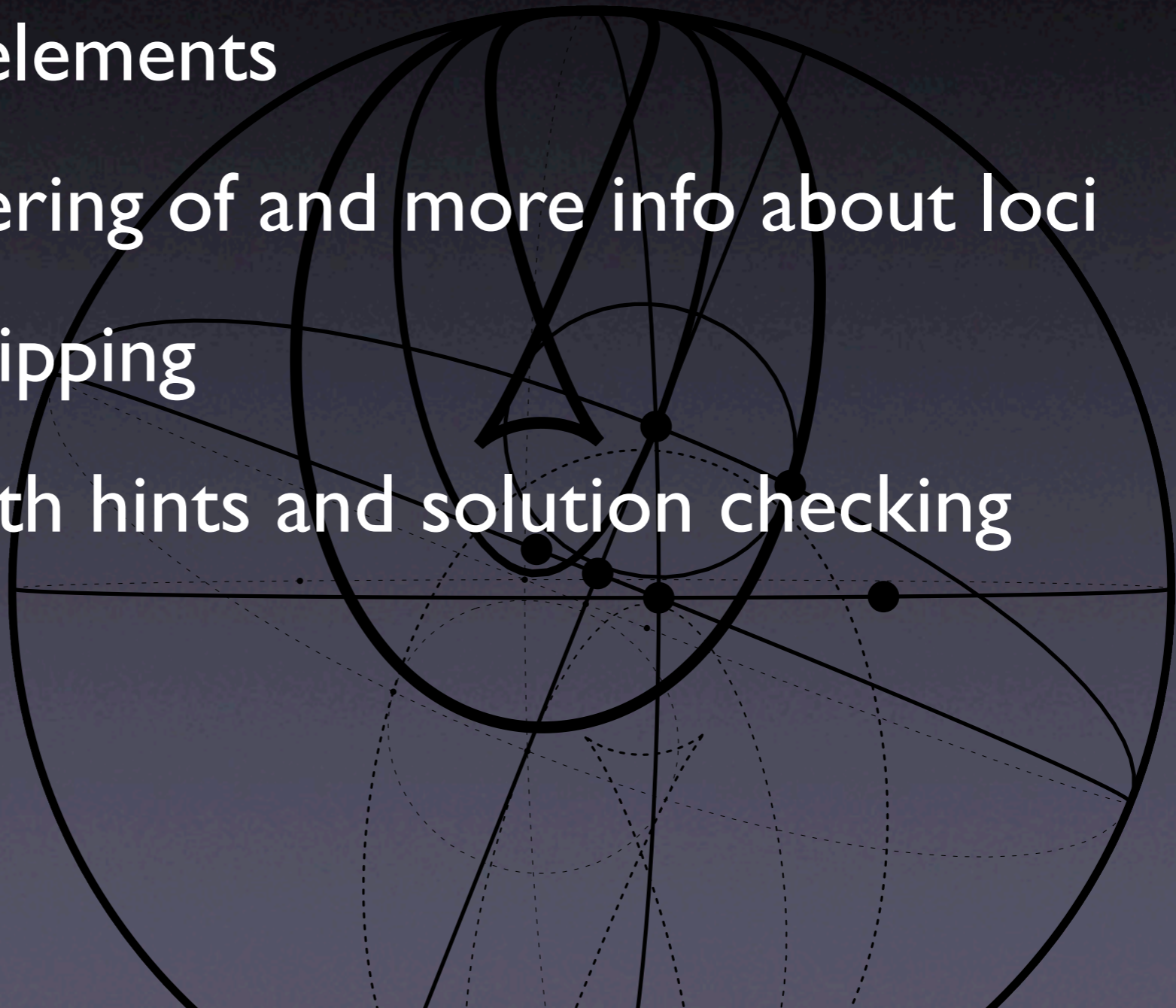
General Strategy

- For any new constructed element, check whether there is another element which is identical (i.e., whose coordinates are equal for any placement of free points)
- If so, re-use the old element as the new one



Benefits from ATP

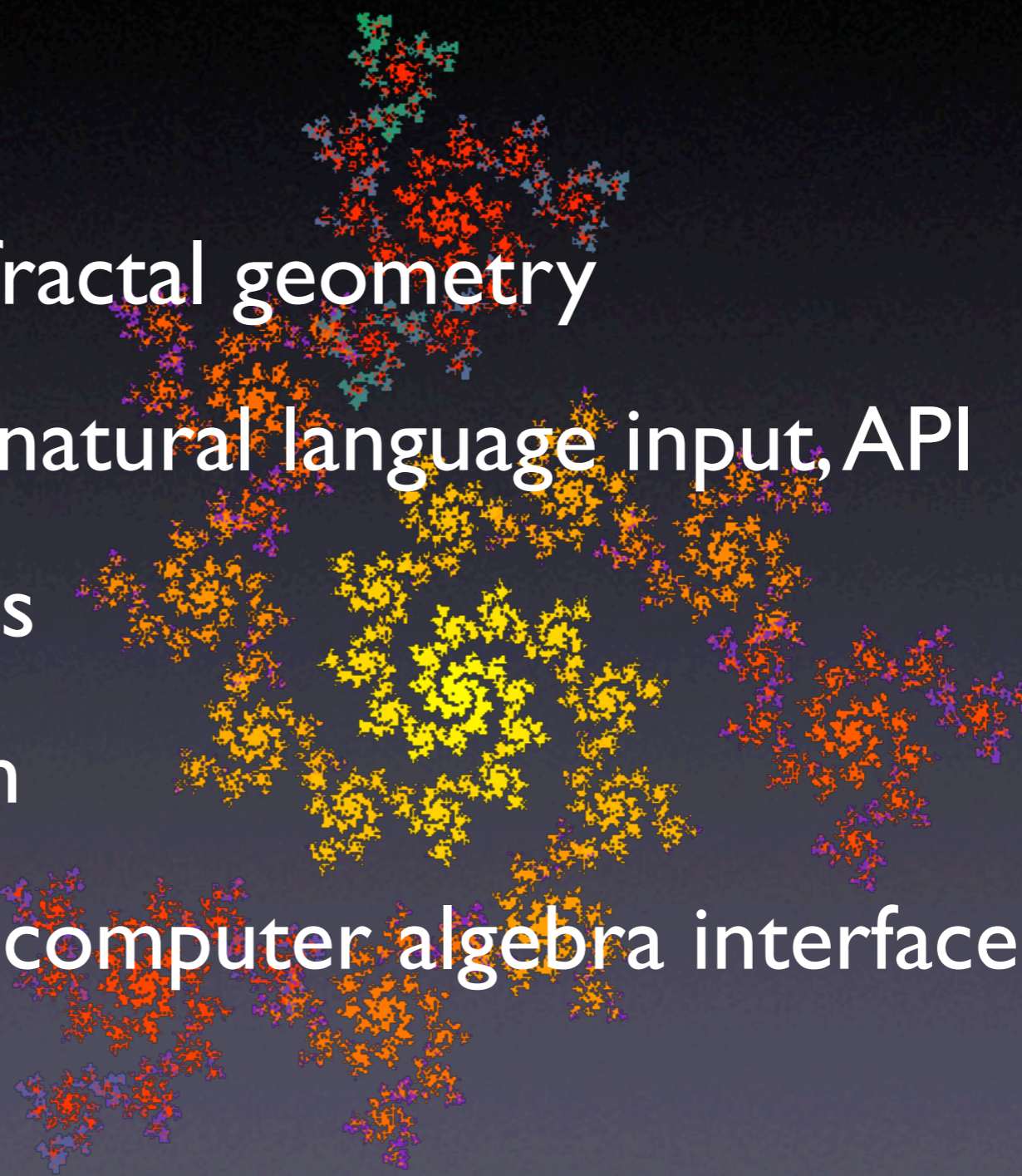
- No double elements
- better rendering of and more info about loci
- automatic clipping
- exercises with hints and solution checking



Cinderella 2: 2001-...

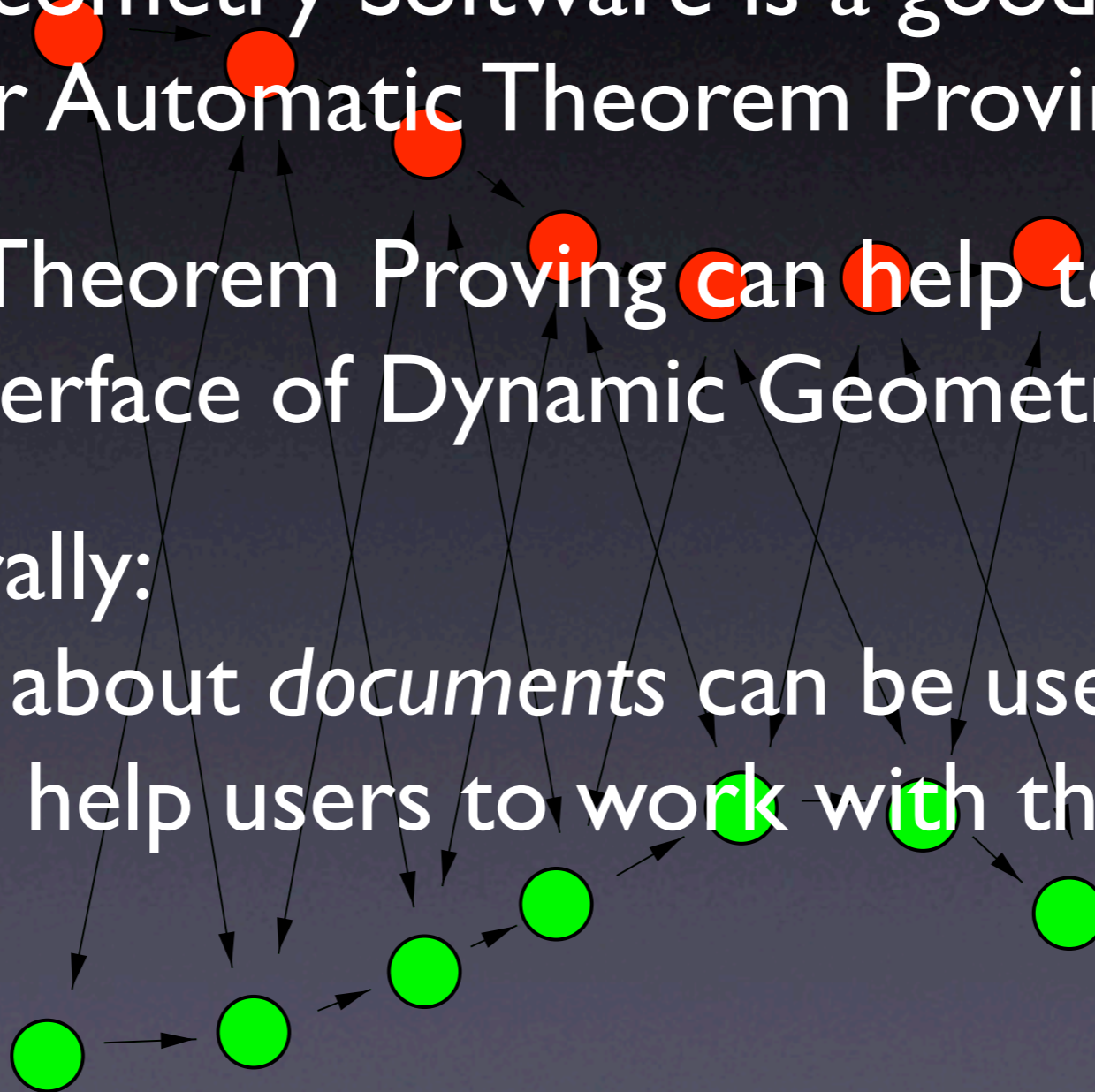
Many extensions:

- Transformations, fractal geometry
- macros, scripting, natural language input, API
- physics simulations
- sketch recognition
- OpenMath-based computer algebra interface



Conclusion

- Dynamic Geometry Software is a good user interface for Automatic Theorem Proving
- Automatic Theorem Proving can help to enhance the user interface of Dynamic Geometry Software
- More generally:
Knowledge about *documents* can be used by the software to help users to work with them



Thank you!

<http://cinderella.de>

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