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COLLABORATIVE DESIGN OF EDUCATIONAL DIGITAL RESOURCES FOR PROMOTING CREATIVE MATHEMATICAL THINKING

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In this paper, we present our experience, while working in the MC Squared project, with the design of educational digital resources aiming at promoting creative mathematical thinking. The resources are produced within an innovative socio-technological environment called "c-book technology" (c for creative) by a community gathering together mathematics teachers, computer scientists and researchers in mathematics education. In this paper, we highlight processes of collaborative design of the "Experimental geometry" c-book resource and we discuss the design choices resulting in the resource affordances to promote creativity in mathematics in terms of personalized non-linear path, constructivist approach, autonomous learning, and meta-cognition based activities, among others.

INTRODUCTION

Promoting creative mathematical thinking (CMT) is a central aim of the European Union by being connected to personal and social empowerment for future citizens (EC, 2006). It is also considered as a highly valued asset in industry (Noss & Hoyles, 2010) and as a prerequisite for meeting current and future economic challenges (National Academies of Science, 2007). CMT is seen as an individual and collective construction of mathematical meanings, norms and uses in novel and useful ways (Sawyer, 2004; Sternberg, 2003), which can be of relevance to a larger (academic, learning, professional, or other) community.

Exploratory and expressive digital media are providing users with access to and potential for engagement with creative mathematical thinking in unprecedented ways (Hoyles & Noss, 2003; Healy & Kynigos, 2010). Yet, new designs are needed to provide new ways of thinking and learning about mathematics and to support learners' engagement with creative mathematical thinking in collectives using dynamic digital media.

The MC Squared project, briefly presented in the next section, looks for new methodologies that would assist designers of digital educational media to explore, identify and bring forth resources stimulating more creative ways of mathematical thinking. The paper then focuses on the design of one such resource, "Experimental geometry" c-book, highlighting the design choices and the resource affordances to foster CMT in its users. Concluding remarks bringing forward factors stimulating creativity in digital resources collaborative design of are proposed in the final section.

THE MC SQUARED PROJECT

The MC Squared project (http://mc2-project.eu/) aims at designing and developing an intelligent computational environment, called c-book technology, to support stakeholders from creative industries involved in the production of media content for educational purposes to engage in collective forms of creative design of appropriate digital media. The c-book technology provides an authorable dynamic environment extending e-book technologies to include diverse dynamic widgets, an authorable data analytics engine and a tool supporting asynchronous collaborative design of educational resources, called "c-books". The project focuses on studying processes of collaborative design of digital media intended to enhance creative mathematical thinking.

CREATIVE MATHEMATICAL THINKING

Based on a literature review and prior studies led by researchers involved in the project related to studying creativity (El-Demerdash, 2010; El-Demerdash & Kortenkamp, 2009; El-Rayashy & Al-Baz, 2000; Haylock 1997; Mann 2006), a definition reflecting our vision of creative mathematical thinking has been adopted. It defines CMT as an intellectual activity generating new mathematical ideas or responses over the known or familiar ones in a non-routine mathematical situation. Drawing on Guilford's (1950) model of divergent thinking, the generation of new ideas shows the abilities of fluency, flexibility, originality/novelty, and elaboration that are defined as follows:

- ***Fluency*** means the student's ability to pose or come up with many mathematical ideas or configurations related to a mathematical problem or situation in a short time.
- ***Flexibility*** refers to the student's ability to vary the approaches or suggest a variety of different methods toward a solution of a mathematical problem or situation.
- ***Originality*** means the student's ability to try novel or unique approaches in solving a mathematical problem or situation.
- ***Elaboration*** is the student's ability to redefine a single mathematical problem or situation to create others, by changing one or more aspects by substituting, combining, adapting, altering, expanding, or rearranging, and then speculating on how this single change would have a ripple effect on other aspects of the problem or the situation at hand.

THE "EXPERIMENTAL GEOMETRY" C-BOOK

The notion of geometric locus of points is the topic of the "Experimental geometry" c-book presented in this section. According to Jareš and Pech (2013), this notion is difficult to grasp at all school levels and technology can be an appropriate media to facilitate its learning. One way is to use dynamic geometry software to "*find the searched locus and state a conjecture*" and a computer algebra system to "*identify the locus equation*" (ibid.).

The challenge in designing this c-book was to exploit c-book technology affordances to propose a comprehensive study of geometric and algebraic characterization of some loci within the c-book. We decided to create activities aiming at studying loci of special points in a triangle. These loci (for example a locus of the orthocenter) are generated by the movement of one vertex of a triangle along a line parallel to the opposite side (see Fig. 1). These are classical problems from the field of geometry of movement that were solved even before the advent of dynamic geometry (Botsch, 1956; Moldenhauer, 2010). Elschenbroich (2001) revisits the problem of locus of the orthocenter in a triangle with a new media, dynamic geometry software. El-Demerdash (2010) uses this example to promote CMT in high school mathematically gifted students.

The c-book description

The c-book invites students to experiment geometric loci generated by intersection points of special lines of a triangle while one of its vertices moves along a line parallel to the opposite side (see Fig.

1). The activity can give rise to a number of various situations, which makes it a rich situation for exploring, conjecturing, experimenting, and proving.

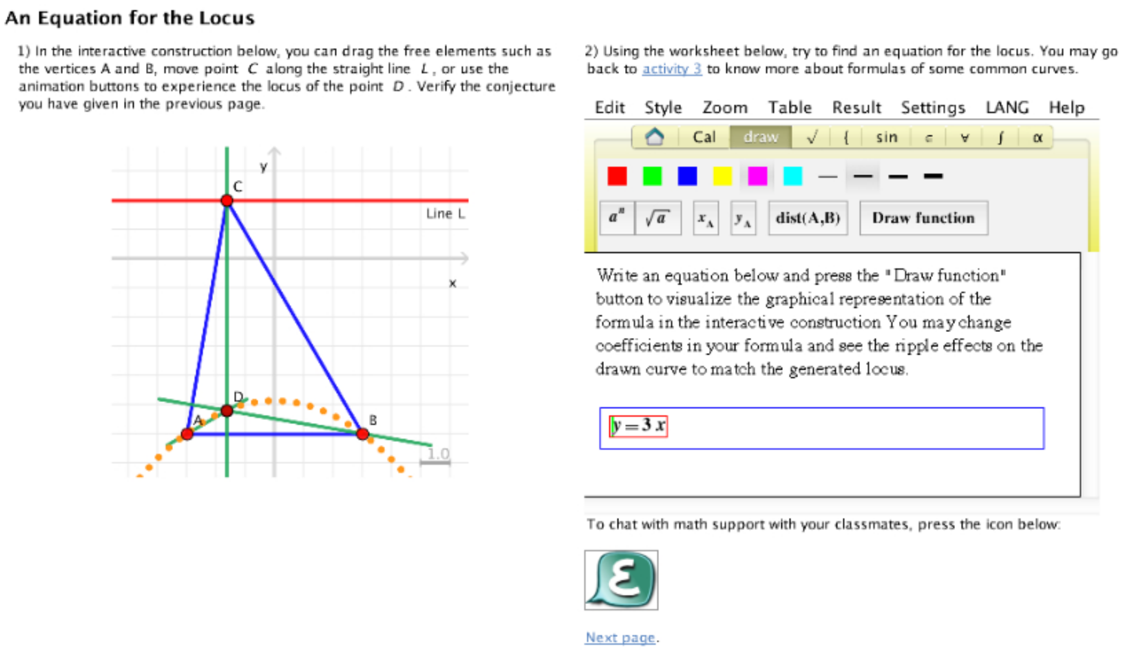

Figure 1: A screenshot of a c-book page showing three widgets: Cinderella (left), EpsilonWriter (top right) and EpsilonChat (bottom right).

The c-book is organized in three sections. The first section proposes the main activity called "Loci of special points of a triangle". It starts by inviting the students to explore, with Cinderella dynamic geometry software[1], the geometric locus of the orthocenter of a triangle while one of its vertices moves along a line parallel to the opposite side (Fig. 2a). The students are asked to explore the situation, formulate a conjecture about the geometric locus of the point D and test the conjecture by visualizing the trace of the point D (Fig. 2b).

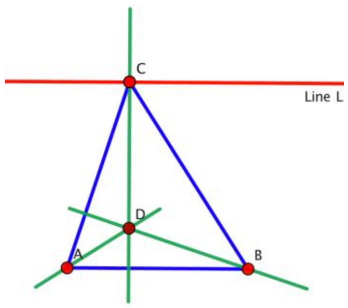

Figure 2a: Geometrical situation proposed with Cinderella (Act. 1, page 1).

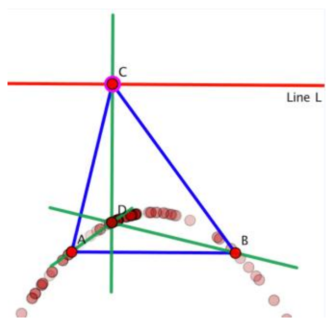

Figure 2b: Visualizing the trace of D while C moves on the red line (Act. 1, page 2).

The students are then asked to find an algebraic formula of the locus, which is a parabola. The formula is to be written with EpsilonWriter[2] application and the interoperability between this widget and Cinderella allows the students to check whether the provided formula fits the locus or not.

[1] http://www.cinderella.de
[2] http://www.epsilonwriter.com

The students are next encouraged to think of, explore, and experiment the geometric loci in other similar situations, such as the locus of the circumcenter (intersection of the perpendicular bisectors), the incenter (intersection of the angle bisectors) or the centroid (intersection of the medians). Other situations can be generated by considering the intersection of two different lines, for example a height and a perpendicular bisector. Twelve such situations can be generated. For each case, one page is devoted offering to the students:

- Cinderella applet with a triangle ABC such that the vertex C moves along a line parallel to the side [AB] and a collection of tools for constructing intersection point, midpoint, line, perpendicular line, angle bisector, locus, as well as the tool for visualizing the trace of a point;
- EpsilonWriter widget enabling a communication with Cinderella;
- EpsilonChat widget enabling remote communication among students.

Section 2 called “The concept of geometric locus” aims at introducing the concept of locus of points. It starts by leading the students to discover the fact that a circle can be characterized as a locus of points that are at the same distance from a given point. The students first experiment a “soft” locus (Healy, 2000; Laborde, 2005) of a point A placed at the distance 6 cm from a given point M (Fig. 3a), and then they verify their conjectures by realizing a “robust” construction of the circle centered at A with a radius 6 cm (Fig. 3b).

(a) (b)

Figure 3: Circle as a locus of points that are at a given distance from a given point: (a) “soft” locus, and (b) “robust” locus.

The next page is constructed in a similar way in order to allow the students to explore perpendicular bisector as a geometric locus of points that are at a same distance from two given points. Finally, the last page proposes a synthesis of these two activities and provides a definition of the concept of geometric locus of points.

The third section, “Algebraic representation of loci”, proposes a guided discovery of algebraic characterization of the main curves that can be generated as loci of points: a circle, a perpendicular bisector and a parabola.

Design choices and rationale

Personalized non-linear path

The c-book is designed to allow students going through it according to their knowledge and interest. They are invited to enter by the main activity in section 1. However, the concept of geometric locus is a prerequisite. In case this knowledge is not acquired yet, or the students need revising it, they can reach the section 2 via an internal hyperlink from various places of the main activity. Similarly, section 3, which allows the students learning about the algebraic characterization of some common curves, is reachable from the main activity. Thus the students can "read" the c-book autonomously, in a non-linear personalized way, depending on their knowledge about geometric or algebraic aspects of loci of points according to their needs.

Promoting creative mathematical thinking

The c-book is designed in a way to support the development of creative mathematical thinking through promoting its four components (fluency, flexibility, originality, and elaboration) among upper secondary school students. First, the main activity is designed in a way to call for students' elaboration: they are invited to modify the initial situation by considering various combinations of special lines in a triangle, whose intersection point generates a locus to explore. Fluency and flexibility are fostered by providing the students with a rich environment in which they can explore geometric situations and related algebraic formulas while benefitting from feedback allowing them to control their actions and verify their conjectures (see learning analytics below). Specific feedback is implemented toward directing students to produce different and varied situations and help them break down their mind fixation by considering yet different configurations, such as two different kinds of special lines in a triangle passing through the movable vertex (e.g., a height intersecting with an angle bisector), and then the intersection of two different lines that do not pass through the movable vertex. The c-book provides the students not only with digital tools enabling them to explore geometric and algebraic aspects of the studied loci separately, but also with a so-called "cross-widget communication" affordances of Cinderella, a dynamic geometry environment, and EpsilonWriter, a dynamic algebra environment, which makes it possible to experimentally discover the algebraic formula that matches the generated locus in a unique way; this feature may contribute to the development of original approaches by the students.

Constructivist approach

The c-book activities in sections 2 and 3 are developed based on the constructivist learning theory practices enabling students to create new experiences and link them to their prior cognitive structure supported with learning opportunities for conjecturing, exploration, explanation, and mathematics communication. The feedback drawing on learning analytics (see below) is designed to allow students solve the proposed activities autonomously and thus construct the target knowledge.

Meta-cognition - Learning by reflecting on one's own action

All c-book sections end up with a meta-cognitive activity that has been designed to encourage students to reflect about their learning and enable them further understand, analyze and control their own cognitive processes. These activities have also been designed to develop students' written mathematical communication skills through the use of EpsilonChat, a widget for communicating mathematics.

Technological development

An outstanding feature of the c-book environment is the fact that it does not only come with a large number of existing widgets in the mathematical context from several different European developer teams, but it also comes with so-called widget factories, one from each of the developer teams allowing authors to generate their own specialized widgets, if they want. The interesting point of this is that all these diverse widgets work perfectly together with the back-end of the environment and they can even collaborate with each other within pages. For example, the dynamic algebra system EpsilonWriter is an interesting tool for manipulating formulas and equations via a unique drag and drop interface (right part of figure 1). But it neither has a built-in function graphing tool nor geometric construction capabilities. These aspects are some of the specialties of the programmable dynamic geometry system Cinderella (left part of figure 1).

Later, when working with the c-book, a student may have produced a reasonable equation for a function within EpsilonWriter, and she can visualize it by using the 'draw' tab. The graph of the function will be shown in the Cinderella construction at the right. For the student, this is visually clear and intuitive; but technically a lot is happening in the background. First, the equation will be sent from the EpsilonWriter software via a standardized protocol to the c-book environment and from there to the Cinderella software, which finally visualizes it as a part of the interactive construction. All this is possible within the c-book player running in a web-browser.

As the example above illustrates, cross-widget communication is a quite powerful feature. In this case, it opens the opportunity for the c-book author to make explicit connections between different representations of a mathematical object: a curve represented as a geometric locus, its formula or equation with the ability to modify it dynamically, and a geometric figure combining both the construction as a locus and the visualization of the curve given by the equation. Within the c-book environment, such opportunities exist in other branches of mathematics as well, e. g., via this mechanism statistics and probability widgets may be connected to geometry, algebra, a number theory widget or even to a logo programming widget, to name just a few more use cases.

Another advantage of the fact that the c-book environment comes with a set of widget factories is that it is easy for a c-book author to create new widgets for the specific needs of the c-book she is currently developing. The quickest way to do this is by adapting existing widgets to specific needs of a currently written c-book. For example, a widget allowing certain geometric constructions by changing the available tools, add some geometric objects, etc. But in addition to this, a c-book author can develop a completely new widget from scratch using one of the widget factories; it will automatically work within the c-book environment via the interfaces implemented on both sides.

Learning analytics and feedback

One of the important aspects in the design of this c-book is to decide which of the student's activities should be logged to a database while she is studying the c-book. There have been many different types of logs implemented in this c-book. These logs enable the teacher to capture the student's path in studying the c-book, e.g., whether the student starts from the c-book main activity, what pages she goes through while studying the c-book, how far she goes through the additional two activities, whether she goes back and forth through the c-book pages and activities and when, whether she uses the provided internal and external hyperlinks to look for further information, how she uses the available hints and how many levels of hints etc.

Moreover, logs were implemented to trace the student's trails or attempts while she is using the provided Cinderella tools to construct a configuration to elaborate the given problem situation: the time the student spends on each page and each activity as an indicator of motivation; the number of student’s trials for each page and each activity of the c-book; the student's use of EpsilonChat as a social aspect of creativity and collaborative work with others whether in pairs or groups.

Two types of feedback are provided to students, while they are studying the c-book to guarantee their smooth move from page to page and switch between the c-book activities: mathematical or educational feedback and technical feedback. Mathematical or educational feedback includes hints and comments oriented toward solving the given problem or developing creative mathematical thinking. This type of feedback is in the form of a message sent in a pop-up window, of a hyperlink or of an internal link. Technical feedback aims at helping students master the available widgets so that technical issues do not become obstacles to the problem solving processes. This type of feedback is in the form of hints or instructions about how to use Cinderella or EpsilonWriter provided tools, or hints regarding the use of cross-communication between the two widgets.

CONCLUSION

The c-book presented in this paper is the result of a collaborative work of a group of designers coming from various professional backgrounds, as the group comprises researchers in mathematics, mathematics education and computer science, as well as educational software developers. Without the synergy among those group members, a number of design choices would have remained in a hypothetical state, namely the technological advances in terms of cross-widget communication and learning analytics features. The design of the c-book has thus become a driving force in the c-book technology development, and in return, the unique c-book technology features enabled the creation of a resource with affordances promoting creative mathematical thinking.

This experience brings to the fore factors stimulating creativity in the collaborative design of digital educational resources. Among these are the following two:

- a variety of designers’ profiles, as pointed out by Fischer (2005), as it encourages the search for novel information and perspectives;
- a close collaboration with software developers, which turned out to be critical for the design and implementation of unique features of the c-book technology resulting in a creative resource. Thus the development of the technology and the educational resources designed with this technology feeds each other.

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