# An activity-theoretic approach to multi-touch tools in early maths learning

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In this article we present an activity theory based framework that can capture the complex situations that arise when modern technology like multi-touch devices are introduced in classroom situations. As these devices are able to cover more activities than traditional, even computer-based, media, we have to accept that they take a larger role in the model of interactions. We reflect this fact by moving the artefact into the center of our observations, leading to an artefact-centric activity theory (ACAT). The theory was developed in the need of analyzing learning environments for primary maths education.

## 1. Multi-touch environments and the ACAT theory

In the last years the human-computer-interface starts to evolve from indirect manipulation via keyboard or mouse to direct manipulation using touch-sensitive interfaces. Through the availability of devices like the iPhone or the iPad we see a rapid adoption of multi-touch-interfaces in all age groups, including primary school students. Multi-touch technology can capture multiple touches on a screen and convert these actions into *events* that can be interpreted by appropriate software. In the simplest case these might be mouse actions like mouse clicks and mouse drags, but because several touches can be combined into *gestures* it is possible for the user to give more information than just translations (that correspond to dragging the mouse) or (x,y) positions (that correspond to clicking the mouse). With two fingers it is easily possible to rotate or scale – or move several objects at once, each in a separate direction, with more fingers more degrees of freedom are available to input almost arbitrary transformations.

It is an obvious step to use such technology for young children in technology enhanced learning environments: (1) The user interface is easy to understand (sometimes even *natural*) and does not add unnecessary complexity to the learning process; (2) The direct manipulation enables children to work with virtual manipulatives directly instead of being mediated through another input device; (3) It is possible to create learning environments using large screens (for example multi-touch-tables) that encourage collaborative learning and communication of the children.

In our example environments we aim at constructivist environments that enable children to move between different representations of numbers. They should be able to work enactively with the virtual manipulatives, presented in iconic form, and – depending on the activity – transfer them automatically either simultaneously or on demand into symbolic representations.

By focussing on direct manipulation via multiple touches we specify an input channel. For the visualization, the output channel, we must choose iconic and symbolic forms that are suitable for representational transfer. Using a multi-touch-enabled programming environment (Richter-Gebert & Kortenkamp, 2012) we are able to specify the connection between the input channel and the output channel. This program, together with the multi-touch hardware creates an artefact that mediates between the actors (the children) and the objects of doing (the virtual manipulatives). The children can only manipulate within the limitations set by this instrument.

The inherent complexity of this environment calls for a theory that is able to guide us in analyzing it. Because multi-touch is the central concept in our setting, we decided to move it into the center of our theory, artefact-centric activity theory (ACAT). It may be strange to move the artefact into the center of a theory that investigates (human) activity – the artefact itself does not have agency and is only mediating. As observed in our experiments (see the videos available at http://cermat.org/acat/videos.html), the artefact changes the way children act drastically and in nonobvious ways. Also, we use Activity Theory not only for analyzing the interaction between subject and object, but in addition for designing the artefact.

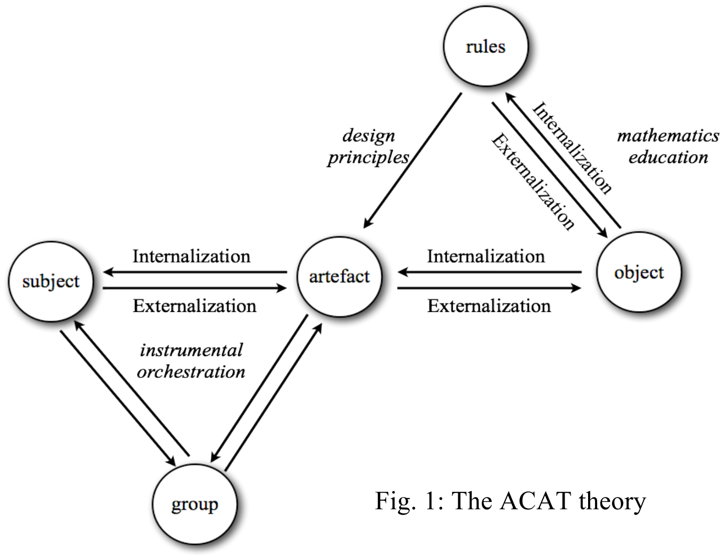


Fig. 1: The ACAT theory

We adapted the activity system diagram of Engeström (1991, p. 31), which is based on *Subject*, *Object,* and *Community* and the three additional mediating means *Mediating Artefacts, Division of Labor,* and *Rules.* We believe that *Rules* in Engeström's sense should also affect the design of the artefact, thus we need a new relation between these two nodes. For clarity we omit the division of labor from the diagram. Because our focus lies on the artefact, we are not considering the relations between the rules and subject, object and community in this article, though they are important for a full activity system (Fig. 1).

The subject and the group are the learners in our teaching/learning situations. It is more difficult to describe the object, in particular because of the dichotomous nature of it: The objects in our environments are numbers, both the current number concept the subject developed and "absolute" number concept. This must not be confused with the external representation of it that is visible on screen. Actually, a desired outcome (in the sense of Engeström) would be a development of the subject's internal representation of numbers into the direction of a mathematically sound one, that is the ideal or absolute object.

The main axis of interaction follows the subject–artefact–object line. A subject –that is, the student– externalizes its concepts regarding an object (in our case: numbers) via an artefact (the multi-touch environment with its virtual manipulatives). The artefact itself externalizes the object through a suitable representation and visualization. The object is encoded into the artefact: the artefact is limited to the object's properties and aspects. The essence of being a number determines the artefact's behavior. Through manipulating the artefact, the student can experience the "numberness" mediated by it. What exactly is meant by "number" depends on what we want to teach (or investigate). It can range from cardinal or ordinal aspects only via combinations of both up to fully developed concepts with all operational aspects (addition, multiplication, inversion of those, etc.). Again, this is also part of the development of an activity system and reflects the genetic method (Wittmann 1974, Führer 1997).

The role of mathematics education (that includes mathematics) is to devise rules that lead to design principles for creating the artefact. An example for such a rule would be that visual representations of numbers by blocks should not be structured arbitrarily but in fives and tens. The structure of tens is obvious from our decimal system. The structure of fives needs some explanation. The so-called "power of five" (Flexer 1986, Krauthausen 1995) enables children to overcome the inability to subitize quantities of more than 5 things. Structuring smaller numbers can be done and is done in other configurations, for example six as a "double three". The power of five, commonly used with "five-frames", develops its potential for numbers up to 100, in particular if used with hundreds charts that indicate the five-frame as well (Fig. 2a). There, it is easy to immediately see that 64 fields are marked. This technique has been described already by Kempinski (1921), who added the 5-structure to a Russian abacus also in the tens (see the left and right markers in Fig. 2b).

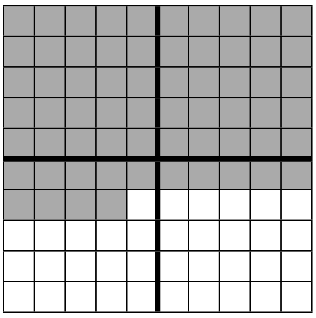

Fig. 2a: You can easily see that 64 fields are grey

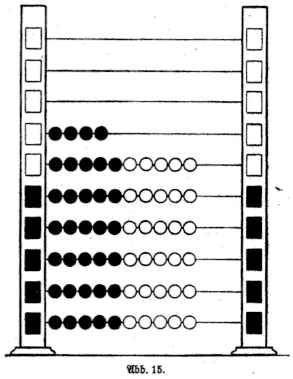


Fig. 2b: Taken from Kempinski (1921)

Describing the creation of rules as an externalization/internalization process is stretching the notion, as the rules are not individuals who could take part in such a process. It should just indicate that the development such rules (and mathematics education as a whole) is similar to the development of acting subjects. Through the rules we define the object formally (externalizing it) and the rules are made to capture the nature of the object in the best way possible (internalization).

Mathematics and mathematics education traditionally create models (in the sense of Klein, see (Klein 1928) and Richter-Gebert (2012)) for abstract objects together with rules to work with them. An abacus as in Fig. 2b is such a model for numbers, and it is possible to add and subtract in this model by moving beads in certain ways. We refer to Thompson (1999) for a discussion of representations, in particular his view on Duval's semiotic registers.

Once such a set of rules is available, we can derive the discipline specific design principles for creating artefacts, that add to the general design principles from psychology and multimedia design (Mayer 2005). In our example that could be that visual hints for blocks of five are added.

A multi-touch-table is particularly well suited for group work as students do not have to take turns with the mouse.

It is hardly possible to fight for an input device – the fingers – if this is available to everybody at the same time. It has been shown that bullying effects (high achieving students take over in an activity by occupying the mouse) and non-subject specific communication that involves agreement discussions about "who is next" are drastically reduced in multi-touch environments, and students engage more in fruitful communication that is close to the topic (Harris et al., 2009, in a study with primary school students doing a planning task for room design; Dohrmann, 2010, in a study with children at age 10-14 at a science exhibition).

The group arrangement that has to be considered is also influenced by the instrumental orchestration (Drijvers et al., 2010) that takes place in the lower left triangle, with the teacher as an additional parameter. We will omit the discussion of instrumental orchestration in this article.

While a multi-touch-artefact seems to be an *enabling* technology as it provides lots of possibilities to work with virtual manipulatives that can act in numerous ways, we must stress that it is also, if not primarily, a *limiting* technology. We want to restrict the students' externalizing actions to support the internalization of specific properties of the objects in consideration. One example for such a restriction would be that the students are not allowed to create stacks of tokens, but have to place them next to each other. In this way, they can subitize or use the "power of five" to quickly recognize quantities. Thus the mediation through the artefact is characterized by restriction and focussing.

## 2. Conceptualizing mathematics knowledge through the creation of rules for design principles and implementation of artefacts in ACAT

For the analysis and further presentation of our theory we will use our object of interest for teaching and learning – numbers and operations – as the object in ACAT. The design of the exemplary artefact using a multi-touch-table is meant to address children of age 5-7. The externalization of the "number" object is handled by visualizing numbers (for example through tokens, symbols or by locations on the number line). The internalization has to be done through programming the multi-touch environment suitably. We will give an example to illustrate this two-way process.

In our case, the "absolute facet" of the object is restricted to the part-whole-concept as a fundamental principle of numbers (see Vergnaud (1979) for more complex models that could be used in earlier or later stages of development of the object). Here, the number 8 –for example– can be seen as the sum of 5 and 3. An artefact that is used for working with numbers and externalizes this object properly should support to split 8 grouped tokens into two parts, say, 5 tokens and 3 tokens that are still in groups. Also, it should keep the operation history, that is, the 5-group and the 3-group must "know" that they originated from the 8.

A student asking for a representational transfer of the iconic representation into symbolic form should receive the information that 8 = 5+3. Thus, the programmed environment has to store this process information, because 8 would be 4+4 or any other decomposition if the student worked differently. The part-whole-concept becomes manifest in the coding of the artefact, if done correctly. The theory does not tell *how* this can be achieved, this is left to the programmer.

Using fingers and finger symbol sets (Brissiaud 1992) is a common strategy in early maths. It is possible to transfer this into rules for designing MTT environments. For example, the "power of five" (see above) tells us to group tokens automatically in groups of five or to offer an easy way to put five tokens at a time on the table (Ladel & Kortenkamp 2009).

We sketch the underlying theory from mathematics education in the following section. Due to the ACAT framework we can pinpoint the essential areas of investigation and see exactly where programming, visualization, design, and the interplay with mathematics education are located.

## 3. AT-based Analysis of Internalization and Externalization Between Subject and Artefact

As an example of a multi-touch learning environment we refer to a prototype we created (Fig. 3). Here we ask students of age 5-7 to place a certain number of virtual tokens on the table as quickly as possible. This number is within the range of up to one hundred tokens. We expect students that have a fully developed number concept to use the structure of 10's to quickly reach a number such as 43 by placing four times 10 tokens and then the remaining 3.

Placing tokens is done by moving one or more fingers from a specially designated "border zone" of the table into the center of the table. If students use a full hand the five tokens associated to the fingers are grouped into a bar of five. These bars of five can easily be recognized and used for keeping the process structure (using a full hand) in a visual form. Students can use this to make use of their number concept for easier (and faster!) placing of the correct number of tokens. Actually, the students can use both hands as well, producing 10 tokens. This is useful when the decimal system is used by the student. Still, we show the ten as two bars of five, as we want the ten tokens to be recognizable quasi-simultaneously (again this uses the "power of five").

The environment can be used either for diagnosis of the development of the number concept of the students or, in a second step, it could be used to enhance their concept using a supplantation approach with automatic structuring aids as described.

In both cases it is helpful to view the work of the students at the table in the light of AT (Kuuti 1996, p. 30): The *activity* in question is solving the task to place 43 (or any other number) of tokens on the table. Students will work on several such goal oriented tasks during a session.

In the framework of Activity Theory we recognize a clear orientation towards the object: Students with a better understanding of numbers and their structure, that is with a fully developed number concept, can work on the activities with less hand movements and more accurately than students without such a competence. In particular, if we ask children to work with as few movements as possible, they may need more time, but this encourages them to use and further develop their number concept.

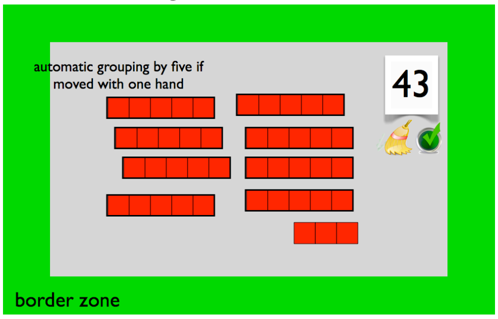

Fig. 3: Example learning environment

The task – to transform the symbolic representation of a number into a cardinal-iconic representation – is created by our desire to understand the development of the students' number concept.

During each activity the student carries out several *actions*. Here, an action is the movement of one or several tokens from the border zone onto the table. Each of these actions consists technically of several touch-drag-release sequences on the table.

For *operations* we distinguish between technical operations and operations of the subject-object interaction in the traditional sense. This honors the fact that the programming and the interaction may develop separately. Every touch-drag-release sequence corresponds to an operation *on the technical side*. Depending on whether these operations are carried out simultaneously or not, it can be judged through the programming of the artefact whether the student uses the "power of five" or not. A student who automatically uses five fingers at once may have gotten to a stage where moving five tokens is already collapsed into a single operation (in the traditional sense). A software that allows for macros (as most DGS do) can offer to combine several operations into one, which happens on the technical side.

When the operations are both spatially and temporarily local – that is, they are carried out at almost the same time and place – the artefact can amplify the structuring approach of the student by creating a bar of five instead of five separate tokens. This detection has to be implemented in the software and must join several technical operations.

From the students' perspective the operations will be collapsed into operations as they will not consciously move single fingers but full hands (or several fingers if they place less than five tokens). Still, from a technical point of view and for the design of the learning environment it is helpful to stay with the granularity of operations, as they are the key to implementing the rules for the multi-touch artefact.

For the implementation we chose Cinderella, a system that is designed to encapsulate mathematical theory and offer this to the designer of tasks (Richter-Gebert & Kortenkamp, 2010). In this software, multi-touch operations are supported via a *touch-locality* mechanism: For each finger (or touch-drag-release sequence) a separate context is created that can be handled individually (Ladel & Kortenkamp, 2011; Richter-Gebert & Kortenkamp, 2012).

Students can realize each task using different actions. We suppose that the choice of these actions depends on the model(s) the children use for representations of quantities. A student using the power-of-five approach already (say, by placing 5+5, 5+5, 5+5, 5+5, 3 to create 43) will benefit from creating tokens with one or two hands at the same time instead of creating them one-by-one or creating them in varying quantities (say, by placing 4, 7, 7, 6, 8, 9, 2 tokens to create 43). These two approaches can be differentiated through the artefact by combining the operations into actions and classifying each of them.

## 4. Conclusion and Outlook

We can take a process-oriented viewpoint on the competence of the children with respect to their number and operations concepts. Instead of assessing the outcome of their work (*are they able to put 43 tokens on the table?*) we assess the process (*how do they put 43 tokens on the table?*). In traditional environments we can understand how developed a certain competence is only by looking at the product. In particular, standard errors can be identified that point out certain misconceptions or under-developed competences. However, for such simple tasks as putting a certain number of tokens on a table the explanatory power of wrong (or right) results is marginal. We hope to use the additional data from operations on the technical side and their combination into groups that match the traditional operations of the subject to enhance the analysis of the video recordings of students working at the table.

The limiting aspect of a virtual manipulative is (positively) influencing the learning process. Without the limitations through the environment students could do anything with the tokens, which we want to prevent. For example, students cannot create stacks of tokens, but they have to place them next to each other, which is visually easier than stacks. This restriction together with an automatic structuring of bars of five could help them to subitize quantities better.

A further extension of the ACAT diagram could involve multiple artefacts. Video studies showed that the operations on the table do not match the operations with the fingers of the children. This suggests that the fingers are another mediating artefact between subject and multi-touch table.

Through the lens of ACAT that places the artefact in the center of attention we can locate the various areas of didactic and pedagogic design that have to be taken into account, and the assessment of learner's progression can move from a product-centric to a process-centric perspective: Instead of just designing tasks and assessing solutions, we can analyze and influence the whole learning process.

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