PROCESS ORIENTATED LEARNING ENVIRONMENTS FOR INTERACTIVE GEOMETRY LESSONS

Andreas Fest[1]

University of Education Schwäbisch Gmünd, Germany

Ulrich Kortenkamp

University of Education Karlsruhe, Germany

Based on the interactive geometry software Cinderella we developed a learning environment to be used for teaching geometry at university level. Following the ideas of the constructivist learning theory the software allows for learning by discovery. The activities are implemented in a process-oriented design. Our software supports both the learning of mathematical processes by the students and analysing their process of learning mathematics. In this paper we focus on a particular learning unit on congruencies and line reflections. By varying the axes of reflection, students can study the reduction theorems for geometric transformations. Different directly given or subliminal hints help the students to understand the mathematical principles behind these theorems. Our tool analyses the student's solution processes automatically and generates feedback on demand to the students. This learning environment can be used in conjunction with the recording of user actions. Our final goal is to be able to analyse learning processes of students using the tool semi-automatically and use this information to improve teaching and learning.

NEW CHALLENGES IN STUDENTS' EDUCATION

Evidence is that teaching mathematics at university level should follow the concepts of a guided discovery learning method (Mayer, 2004). Students deal with certain mathematical problems and by developing their own solution strategies they should discover deeper mathematical relations and structure. The teacher's role in such a self-exploring learning environment is to support the learning of the students. Nevertheless he or she cannot accompany the whole learning process of all students individually. The use of new media in the mathematical education can support the teacher in this task.

According to the ideas of the constructivist learning theory suitable learning software should open the door for individual learning pathes to the students (Schulmeister 2007). The individual learning process of the students should be aided by an intelligent feedback system that offers more information than just a "right" or "wrong". Especially in mathematics there often is an innumerable amount of conceivable solution strategies, and they cannot be validated automatically. But

[1] supported by Bundesministerium für Bildung und Forschung, Germany

fortunately this is not necessary in our approach: Most students follow one of a few common solution strategies. Even the mistakes students make are mostly of the same type, as they are based on standard misconceptions. By removing these automatically detectable solutions from the analysis, usually only a few special cases remain that can be handled by manual inspection of the teacher. This concept is called semi-automated assessment (Bescherer, Kortenkamp, Müller, & Spannagel, in press) and can be integrated in many learning environment, in particular for mathematics. While the students get individual feedback on standard solutions or mistakes directly from the software, the teacher is notified of those non-standard solutions. As a consequence, the teacher is relieved from the discussion of common standard solutions. He or she gains more time to analyse interesting exceptional solutions and to discuss unusual problems with the students. Also, using the statistical data about the occurrence of certain standard solutions, he or she can focus on the most common problems, if necessary.

TYPES OF FEEDBACK

One important benefit of using interactive media in education is the possible presence of intelligent feedback systems. As opposed to tutorials given by human teachers, the computer is able to give continuous feedback on the students' learning processes. Hence, the theory of feedback is an important area in the theory of teaching with hypermedia. A survey on the different types of is given i.e. by Schulmeister (2007).

To characterize the kinds of feedback given by an interactive learning environment, we distinguish at least between the following aspects:

Timing of feedback. Feedback can be given immediately after each user action, delayed, on demand, or after completing a session (cf. Cohen , 1985).

Information content of feedback. The information the user gets about his or her lerning process can vary between "verification feedback" and "elaboration feedback" (Pridemor*e* & Klein, 1991). While the "verification feedback" just informs about the correctness of a solution, an "elaboration feedback" presents the correct solution and an explanation. Sometimes it may also be sensible to show only partial solutions.

Presentation of feedback. Feedback can be given visually or acousticly, and in iconic/graphic or textual form. Visual feedback can be presented animated or statically. Park and Gittleman (1992) claim that animated visual feedback can be superior to any static type of feedback.

SEMI-AUTOMATICAL FEEDBACK IN INTERACTIVE GEOMETRY

Bescherer and Spannagel (2009) developed a design pattern for (semi-)automated feedback on demand in educational mathematical software. They propose to record and analyse the whole learning process of the students. Recording and analysing can either be done by a generic tool or directly by the learning software itself. Process-oriented feedback should be available on demand by students. Additionally, the teacher should have access to all recorded processes to be able to select interesting processes for presentation and discussion in the course.

We implemented this pattern in the learning unit on congruencies in the Euclidean plane. The software consists of several Java applets[2] embedded into a sequence of HTML documents.

A learning unit on congruencies and line reflections

The learning unit should deepen the students' understanding of planar congruencies and their compositions. The central aspect is the reduction theorem that claims that each congruency can be represented as the composition of at most three line reflections. Consequently, the learning unit uses line reflections as generators.

The unit consists of five parts, described below, that can be worked on separately or in sequence. Each part offers a set of predefined examples. Additionally, the students can create their own examples by selecting a sequence of lines and defining a new congruency as the composition of the corresponding line reflections. The software then detects the type and main parameters of the user defined congruency.

Part 1: Experience congruencies. The student can explore the effects of different congruencies on a given triangle by observing it and its transformation image. The position of the preimage triangle can be varied by dragging, rotating and turning it. The same holds for the axes of the defining line reflections.

Part 2: Operate congruencies by hand. Given a congruency, which is shown symbolically in textual description or as a algebraic term, and a triangle, the student should determine the correct position of the triangles image manually. This exercise is meant to deepen the "feeling" for the effects of the different transformation types.

Part 3: The inverse congruency. If a congruency is represented as a composition of line reflections, the inverse of this transformation results from reversing the algebraic transformation term. Students can explore this coherency by finding the preimage of a triangle under a given congruency.

Part 4: Determine the transformation. By using geometric construction tools the student should determine a sequence of axes such that the composition of the corresponding line reflections maps a given triangle on a given image. Again, the first triangle can be varied.

Part 5: Simplify the transformation term. A given algebraic term describing the composition of different congruencies should be reduced according to the reduction theorem. For that purpose, different geometric and algebraic operations can be performed. Finally, the type and the main parameters of the presented congruency should be identified from the reduced transformation term.

[2] Based on Cinderella 2.1 (Richter-Gebert & Kortenkamp, 2006)

Implementation of interactive feedback

All activities come include appropriate feedback mechanisms. According to the different exercises various types of feedback on demand is implemented. Due to the limitations of possible solutions in the first three exercises, a fully automated feedback can be provided. By contrast, the next two exercises open a wide variety of possible solution processes. Here, we implemented a semi-automated feedback system to review standard solutions or standard mistakes, and to recognize exceptional solution processes.

Obviously, visual feedback is particularly suited in a geometric learning environment. Hence we implemented animated visual feedback in every case where it gives enough information. For example, in Parts 2 and 3 the correctness of the student's answer is signalled by a temporary change of the colour of the solution triangle to green respectively red. The correct position of the asked-for triangle can be displayed on student's demand, supplemented with the trace of all intermediate steps of the repeated transformation of the triangle according to the composition of congruencies.

In cases where visual feedback is unsuitable, we implemented other methods. For example, students are not required to perform any action that could reveal their answer. Consequently, we implemented a textual elaboration feedback, telling the students the correct type of the congruency on demand.

For the last and most complex part we implemented algebraic and numerical control algorithms to generate a feedback as informative as possible. The algebraic checking algorithms allow detection of both correctness and type of term transformations and are a basis for analysing the student's solution process. But not all possible transformation steps can be detected and reviewed by the algebraic check algorithms. In that case a numerical correctness check will be executed. Since this check cannot distinguish between the different types of term transformation, the numerical check is the starting point to initiate a manual analysis of the solution process by the teacher.

Recording of students' learning processes

To implement teacher's access to all learning processes, all high-level user interaction events are recorded by the applets and can be submitted to a server via an internet connection. Such a recording can be stored and analysed retrospectively. Additionally, an offline version of the learning unit can be used together with capture & replay tools like CleverPHL (Schroeder & Spannagel, 2006). The collected data can be used for a deeper analysis of the students's learning processes and unusual solutions or untypical mistakes can be filtered out automatically. See Spannagel & Kortenkamp (2009) in this issue.

CONCLUSION

The teaching unit on congruencies and line reflections is a prototype implementation of the "Feedback on demand" pattern. We demonstrated how this pattern can be realized for a concrete task in geometry teaching.

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