Conceptualizing preschool teachers' knowledge and self-efficacy for teaching mathematics: The CAMTE framework[1]

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1. Introduction

Concern for preschool mathematics education may be seen in the rise of national curricula in various countries which now make specific and sometimes mandatory recommendations for including mathematics as part of the preschool program. For example, in England, the non-statutory Practice Guidance for the Early Years Foundation Stage (2008) suggests ways of fostering children's mathematical knowledge from 0–5 years. In Israel, the National Mathematics Preschool Curriculum (INMPC, 2008) is mandatory and contains specific guidelines and aims for children from 3-6 years. With new standards come new demands for teachers and the necessity for providing teachers with the tools to meet those demands.

This paper describes a professional development program for preschool teachers, which aimed to promote teachers' knowledge necessary for teaching mathematics in preschool. The paper is divided into two sections. The first section introduces the *Cognitive Affective Mathematics Teacher Education (CAMTE)* framework, used in planning and implementing the program. Acknowledging that knowledge and beliefs are interrelated and that both affect teachers' proficiency (Pehkonen & Törner, 1999; Schoenfeld, 1992; Schoenfeld & Kilpatrick, 2008; Törner, 2002), the framework and program take into consideration teachers' knowledge as well as self-efficacy beliefs to teach mathematics in preschool. The second part of the paper provides segments of the program that illustrate how the above framework was used to plan and implement the professional development program.

2. The *Cognitive Affective Mathematics Teacher Education (CAMTE)* Framework

The framework used in our program takes into account teachers' knowledge and their related self-efficacy beliefs. In this section we present the theoretical framework which guides our program and our investigation of teachers' knowledge and self-

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efficacy beliefs. The section begins with a discussion of teachers' knowledge for teaching and continues with a review of self-efficacy. We then present the model of the framework and how it relates to preschool teachers' knowledge and self-efficacy for teaching mathematics.

2.1 Teachers' knowledge for teaching

In framing the mathematical knowledge preschool teachers need for teaching, we draw on Shulman (1986) who identified subject-matter knowledge (SMK) and pedagogical content knowledge (PCK) as two major components of teachers' knowledge necessary for teaching. In our previous work (Tabach, et al., 2010), we found it useful to differentiate between two components of teachers' SMK: being able to produce solutions, strategies, and explanations and being able to evaluate given solutions, strategies, and explanations. Thus our framework takes into consideration both of these aspects of SMK. Regarding PCK, we draw on the works of Ball and her colleagues (Ball, Thames, & Phelps, 2008) who refined Shulman's theory and differentiated between two aspects of PCK: knowledge of content and students (KCS) and knowledge of content and teaching (KCT). KCS is "knowledge that combines knowing about students and knowing about mathematics" whereas KCT "combines knowing about teaching and knowing about mathematics" (Ball, Thames, & Phelps, 2008, p. 401).

Within the domain of number, preschool teachers' SMK includes, for example, knowledge about counting, operations, and a variety of possible ways and methods of rationally examining and explaining one's solutions. Teachers' KCS includes, for example, knowledge of young children's non-conservation of number (Piaget & Inhelder, 1958). Within geometry, preschool teachers' SMK includes knowledge of defining geometrical concepts and identifying various examples and nonexamples of two and three-dimensional figures (solids) as well as ways of justifying this identification. Teachers' KCS includes knowledge of which examples and nonexamples children intuitively recognize as such (Tsamir, Tirosh, & Levenson, 2008a) as well as knowledge of children's commonly held concept images and concept definitions for geometrical figures (Tall & Vinner, 1981). In both domains, KCT includes knowledge of designing and assessing different tasks, affording students multiple paths to understanding.

2.2 Self-efficacy

The framework used in our program also draws on Bandura's (1986) social cognitive theory which takes into consideration the relationship between psychodynamic and behaviouristic influences, as well as personal beliefs and self-perception, when explaining human behaviour. Thus, besides investigating preschool teacher's knowledge it is important to also relate to their self-efficacy. Bandura defined self-efficacy as "people's judgments of their capabilities to organize and execute a course of action required to attain designated types of performances" (1986, p. 391). Hackett and Betz (1989) defined mathematics self-efficacy as “a situational or problem-specific assessment of an individual’s confidence in her or his ability to successfully perform or accomplish a particular [mathematics] task or problem” (p.262). The *CAMTE* framework takes into consideration teachers' mathematics self-efficacy as well as their pedagogical-mathematics self-efficacy, i.e. their self-efficacy related to the pedagogy of teaching mathematics. Teacher self-efficacy has been related to a variety of teacher classroom behaviors that affect the teacher’s effort in teaching, and his or her persistence and resilience in the face of difficulties with students (Ashton & Webb, 1986). Studies report that teachers with a high sense of self-efficacy are more enthusiastic in teaching (Allinder, 1994) and are more committed to teaching (Coladarci, 1992).

The design of our program and the accompanying study was based on the framework presented in the following 8-cell knowledge and self-efficacy matrix (see Table 1). In cells 1-4, and in cells 5-8, we address teachers' knowledge and self-efficacy respectively.

	Subject-matter		Pedagogical-content	
	Solving	Evaluating	Students	Tasks
Knowledge	Cell 1	Cell 2	Cell 3	Cell 4
Self-efficacy	Cell 5	Cell 6	Cell 7	Cell 8

Table 1: The Cognitive Affective Mathematics Teacher Education Framework

Below, we illustrate the different cells of the framework within the domain of number concepts focusing on teachers’ knowledge for teaching counting and enumeration. Counting refers to saying the number words in the proper order and consistently, and knowing the principles and patterns in the number system as coded in in one's natural language (Baroody, 1987). Enumeration relates to counting the number of objects in a set. Gelman and Gallistel (1978) outlined five principles of counting objects. The three “how-to-count" principles include the one-to-one principle, the stable-order principle, and the cardinal principle. Two "what-to-count" principles include the abstraction principle, and the order-irrelevance principle. For each cell we offer specific examples.

Cell 1: producing solutions: e.g. compare the number of elements in two sets using a variety of strategies; count the following large collection of items using a variety of strategies.

Cell 2: evaluating solutions: e.g. evaluate the following strategies for comparing the number of elements in two sets; evaluate the following justifications for why one set has more elements than another set.

Cell 3: knowledge of students’ mathematical ways of thinking and common misconceptions: e.g. which number symbols are more difficult for children to learn; what are children’s common mistakes related to the counting sequence.

Cell 4: designing and evaluating tasks: e.g. tasks which may be used to foster children’s acceptance of the one-to-one principle necessary for enumeration; tasks which may be used to assess children’s counting and enumeration skills.

Cell 5: mathematics self-efficacy related to producing solutions: e.g. teachers' beliefs regarding their ability to count a relatively large collection of items in multiple ways.

Cell 6: mathematics self-efficacy related to evaluating solutions: e.g. teachers' beliefs in their ability to evaluate various strategies for counting.

Cell 7: pedagogical-mathematics self-efficacy related to children's conceptions: e.g. teachers' beliefs in their ability to identify children's common mistakes related to counting.

Cell 8: pedagogical-mathematics self-efficacy related to designing and evaluating tasks: e.g. teachers' beliefs in their ability to design tasks that may be used to promote children’s correct and efficient counting strategies.

The above framework was used to plan and implement our professional development program as well as to study preschool teachers' knowledge and self-efficacy to teach mathematics in preschool. The framework served as an organizing tool as well as a set of checks and balances. We use it to ask ourselves – What do preschool teachers need to know when teaching mathematics in preschool? Are we paying attention to different types of knowledge? Are we devoting time to each of the different elements signified by the different cells? And yet, although each cell specifically focuses on a different piece of the knowledge and self-efficacy puzzle, promoting the different elements are often intertwined. This is illustrated in the next section as we describe segments of our program.

3. Using the *CAMTE* to plan and implement professional development

Cells 1 and 2 of the *CAMTE* framework focus on teachers' knowledge of solving mathematical tasks as well as evaluating solutions to mathematical tasks. In order to plan mathematical activities within the domain of number concepts to be implemented with teachers, we reviewed the Israel National Mathematical Preschool Curriculum (INMPC, 2008) as well as curricula from other countries. Several mathematics curricula around the world mention set comparison as part of the preschool curricula. For example, in England, the Practice Guidance for the Early Years Foundation Stage (2008) suggests that young children engage in activities such as matching and comparing the number of objects in two sets, including the empty set as an introduction to the concept of zero. The INMPC, 2008) states that preschool children should be able to compare the number of objects in different sets as well as be able to divide one set of objects into an equal number of objects. Knowledge of mathematics also includes knowledge of mathematical processes such as reasoning, communication, and problem solving. Within the context of counting and enumeration, this may include being able to count the number of items in a set using different strategies and being able to evaluate which of those strategies are most appropriate for a given situation.

In one of our programs (described in more detail in Tirosh, Tsamir, Levenson, & Tabach, 2011) teaches were given the following task to solve:

Here are two sets A and B:

$$A = \{1, t, \alpha\} \quad B = \{7, w\}$$

Is the number of elements in sets A and B equal? Yes / No

How did you reach this conclusion?

It may seem that the question above is artificial and that perhaps an everyday context would be more appropriate for preschool teachers. However, a more concrete task might not have afforded the teachers the same opportunity to experience children's difficulties and to discuss the abstraction of number concepts. In addition, familiarizing teachers with set notation and the use of brackets allowed us later on to discuss sets in more general terms, rather than always refer to specific examples. We also note that in Hebrew, the word for a set of concrete objects is the same word used for the mathematical notion of sets. Thus, although it may seem to the reader that the language of sets may seem formal for preschool teachers, it is actually quite a familiar term.

The sets in the above example have an equal number of elements. The explanation given by all teachers referred to counting the elements in both sets. As one teacher wrote:

> Set A has three elements and set B has two elements. In order to determine if two sets have the same number of elements, you need to count the number of elements in each set.

In order to encourage the use of additional strategies, teachers were then presented with an additional question (see question 2 below).

This question provided an additional strategy for comparing the number of elements in two sets. In fact, it is not possible to count the paired dancers of Question 2. Instead, one-to-one correspondence between men and women indicated equivalence between the two sets.

> 2. At a dance party all the students danced in couples, a boy and a girl in each couple. No pupils were left without a partner.
>
>   Z = {The boys}   W = {The girls}
>
>   Is the number of elements in set Z equal to the number of elements in set W? Yes / No
>
>   How did you reach this conclusion?

In our program, tasks play a central role in developing teachers' knowledge. Above, we illustrated how engaging teachers with tasks facilitated the development of their knowledge related to Cells 1 and 2 of the framework. Also important is developing teachers' knowledge of how students may interact with tasks (Cell 3 of the framework) and developing teachers' knowledge of designing tasks to be implemented with children (Cell 4). Often, these last two elements are inter-related. This is illustrated in the section below.

In the following excerpts we describe a session where preschool teachers discuss tasks that may be used to assess children's knowledge of enumeration.

> I: According to the curriculum guidelines, by the end of kindergarten a child should be able to count 30 objects. In order to assess if a child can count we first need to ask him to count without giving him objects. If he doesn't know the number sequence, he will not be able to count objects. Now, how many objects should I place before the child to count?
>
> (Teachers offer different amounts.)
>
> I: I probably shouldn't start with 30 because he may know how to count but the large amount can make it difficult. How about 10 items? Why isn't it a good idea to start with 10?

T1: It's a large number.

T2: Because we have 10 fingers.

T3: Automatically, they say 10.

I: Right. How about 8? (The instructor places 8 identical bottle caps on the table.) Many times, a child expects there to be 10 so he won't necessarily take care to point to each item one a time. Instead, he might run his finger quickly over the items saying the number from 1 to 10. So, if we place 10 items in front of the child, we may not be able to discern if he understands the one-to-one principle. So, 10 is not a good number for an assessment task. A good assessment task tests one principle or one piece of knowledge, at a time.

In the above segment, we see how developing teachers' knowledge of tasks is intertwined with their knowledge of children's conceptions. In designing an assessment task, one needs to take into account possible children's strategies and how the specific task may encourage or discourage specific strategies. Knowing that ten is a benchmark number for children and that children may automatically count until ten regardless of the number of actual items that need to be counted, guides the instructor and teachers in choosing a different amount of items. It is also important to consider the types of items to be counted. The instructor points out that if the items are of two colors, the child may count each color group separately. She recommends starting by having the children count a set of homogenous objects and then afterwards, check what happens with heterogeneous items.

In planning our program, care was taken at each step to consider self-efficacy. When addressing the issue of self-efficacy we considered both teachers' mathematics self-efficacy (Cells 5 and 6) as well as their pedagogical-mathematics self-efficacy (Cells 7 and 8). Self-efficacy beliefs are not only domain specific (e.g. mathematics, history, science) and content specific (e.g. within the domain of mathematics there is numeracy, patterns, geometry, etc.), but may well be task specific (e.g. what is the child asked to do) and situation specific (e.g. is the task implemented in class, outside, individually, in a group) (Pajares, 1996; Zimmerman, 2000). Taking this into consideration, we began each new topic with a series of self-efficacy questions related to specific tasks. For example, within the domain of geometry, questions related to Cell 5 included: denote on a scale from 1-4 your ability to define a triangle, to identify a figure as a triangle, to identify a figure as a non-triangle. Questions related to Cell 6 included: denote on a scale of 1-4 your ability to evaluate a definition of a triangle, to evaluate an explanation for why a given figure is or is not a triangle.. Within the context of number concepts, questions related to Cell 7 included: on a scale of 1-4 relate your ability to point to number symbols which children find difficult to learn, to point to arrangements of items which children find difficult to count, to point to numbers which children find difficult to say which number comes next. Questions related to Cell 8 and number concepts included: denote on a scale of 1-4 your ability to design tasks which can assess children's knowledge of counting till 30, to design tasks which can promote children's knowledge of the number symbols from 0 – 9, to promote children's knowledge of the number combinations for seven.

What does it mean to work with teachers taking into consideration their self-efficacy? To begin with, we acknowledge that our goal is not only for teachers to have a high self-efficacy, but that this self-efficacy should correspond to actual performance. If teachers have a high self-efficacy to perform some teaching task, but in reality, cannot perform the teaching task, we have missed our goal. On the other hand, if teachers have a low self-efficacy, despite their being very capable, then we have again missed our goal.

How to achieve our goal? A crucial step is having teachers recognize when they do not know something or do not have the ability to perform some task. Without this step, teachers may not feel the need to actively participate in the program. In a previous study (Tirosh & Tsamir, 2010), we reported on a preschool teacher who began our program with a high mathematics as well as a high pedagogical-mathematics self-efficacy related to teaching triangles. We showed that as the intervention progressed, the teacher came to realize how much she really did not know, which in turn caused her self-efficacy to fall. By the end of the program, however, her self-efficacy as well as her SMK and PCK for teaching triangles, rose. This study indicated that it may first be necessary for a teacher's self-efficacy to decrease before we can achieve our final goal.

Another important aspect of our program related to the issue of self-efficacy is making explicit to teachers the goal of a lesson. In order for teachers to correctly assess their ability to perform a task, we feel it is imperative that during instruction, they are aware of the new abilities they are forming. Thus, for example, at the start of a lesson related to counting tasks, the instructor announced:

> Today we will build assessment tasks. The idea is to build a task that can test and analyze the child's way of thinking and to discuss together possible children's answers in order to know how to continue working with the child… In order to focus on children's ways of thinking, each one of you will implement the same task in their class and bring the results here so we can discuss together the results. We are talking about assessment tasks (the instructor writes this on the board). We do not always have the time to do this in class but it is very important so that we can know where the individual child stands.

During the lesson, the instructor reminded the teachers of the curriculum guidelines and what the child should know by the end of kindergarten. When discussing different number combinations that make up the number five, the instructor suggests a task that involves using five items which are exactly the same. One of the teachers suggested that it would be better if the items were not exactly the same in order to help children see the different ways of building five. At that point the instructor reminds her, "We do not want to intervene. We want to see what strategies the children will employ." In other words, the instructor reminds the teachers that at times it is important to stand back and observe. Being aware of students' strategies, conceptions, and misconceptions can then help us in planning effective teaching tasks.

Discussing the issue of self-efficacy, up front, what it means for them as teachers, was also part of the program. The instructors discussed with the preschool teachers the difference between being able to perceive their students' abilities and being able to perceive their own abilities. As the instructor said

> We discussed together how we see ourselves… Self-perception, in general, is how we think of ourselves… Self-efficacy is how we perceive our ability to fulfill tasks… I judge my own ability. Of course, it is even more important to make connections between how I see myself and how I really am.

The instructors went on to discuss the importance of giving children positive feedback and having them build a positive sense of self-efficacy. However, hand-in-hand, we must provide real opportunities for them to succeed so that their positive self-efficacy will be real and based on their own experiences. According to Bandura (1986) performance attainments are an important source of self-efficacy; successes raise self-efficacy while repeated failures lower them. Thus, when children are given opportunities for success they are apt to believe more positively in their abilities than those without these experiences. On the other hand, self-efficacy beliefs also impact on performance. Thus, we may say that self-efficacy beliefs and performance have a reciprocal relationship. Finally, appropriate feedback may be especially important for young children who have little experiences of their own to reference. To summarize, by discussing children's self-efficacy, as well as their own self-efficacy, teachers became aware of the necessity to have and promote a positive self-efficacy, but one that is backed up by corresponding actual task performance.

4. Conclusion

The preschool teacher plays a major role in fostering children's mathematical abilities. "It is up to her to devote attention both to planned mathematical activities as well as mathematical activities which may spontaneously arise in the class and to pay attention to the mathematical development of the children" (INMPC, 2008, p. 8). Being able to plan appropriate mathematical activities requires knowledge of mathematics as well as knowledge of students and tasks. However, if we want teachers to recognize opportunities for learning mathematics and make the most of these opportunities, they need to be on the lookout for such opportunities, to be proactive. A high self-efficacy for teaching mathematics, based on actual experiences of solving mathematical problems and evaluating possible solutions, based on effectively implementing planned tasks with children and seeing the results of their work with children, can help foster the positive drive we ask of our teachers. That is our aim as teacher educators – to promote a high self-efficacy for teaching mathematics in preschool which corresponds to a high level of knowledge for teaching mathematics in preschool.

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