It is quite confusing isn't it?

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Abstract

The focus of this paper was to identify and discuss the ways in which a specialist mathematics teacher, considered locally to be an ambassador for the subject, presented key conceptual knowledge about partitioning to young children (5-6 years old) during a whole class activity. The case study reported here examined observation and interview data to identify the teacher's rationale for her strategies, methods and resources used to support her teaching objectives. When analysing the data unexpected pedagogical questions arose regarding the teacher's choice of resources and use of language to support children's mathematical thinking and learning. The structure of the session was very confusing to the learning.

Key words: resources (manipulatives), language, partitioning, place value

The understanding of subject knowledge to teach mathematics is not disputed. It has been seen, however, through research that the effectiveness of teachers in teaching mathematics is not only due to the depth of their knowledge but making connections within the subject (Askew et al,1997). Ma (1999) argues that 'it was important for teachers to have a deep knowledge of the mathematics *at the level they were teaching* rather than having knowledge of advanced mathematics'.

Ball's (1988) view is that not only should mathematics be revisited but also pre service teachers may need to unlearn what they know about teaching and learning of mathematics. Goulding, Rowland and Barber (2002) in their portrait of Frances, an early years specialist, revealed that successful teaching of mathematics is not guaranteed by subject knowledge alone. Frances was lacking in confidence and although she knew the theory behind the teaching of subtraction, due to problems with management, she resorted to time filling activities.

With these notions outlined above in mind, we decided to investigate how an experienced teacher with good subject knowledge, and considered to be locally an effective mathematics teacher, would use resources to teach the abstract notion of place value. In particular, we wanted to examine the way in which she would use any resources (manipulatives) to support young children's mathematical learning. It was considered to be an effective way to eliminate any issues related to working with weaker teachers of mathematics, so that the focus would be on the ways in which mathematical thinking was developed through the use of resources, rather than investigating a teacher's lack of confidence in teaching mathematical concepts.

Theoretical background

In 1986, Lee Shulman et al. introduced 'pedagogical content knowledge'. This term called attention to a special kind of teacher knowledge that links content and pedagogy. 'Pedagogical content knowledge – representations of particular topics and how children tend to interpret them , for example , or ideas or procedures with which students often have difficulty – describes a unique subject- specific body of knowledge that highlights the close interweaving of subject matter and pedagogy in teaching' (Ball and Bass 2000).

Teaching place value needs a lot of pedagogical subject knowledge to unpick the stages that children go through. Askew (2011) discusses that '...the learner cannot go directly from not knowing about place value to knowing about it. Teachers provide tools and artifacts that mediate this, through which the learning is assumed to be enabled'. Part of that mediation has to come through appropriate use of language and specific vocabulary which enculturates children in learning to think and speak mathematically (Lerman, 2001). Thompson (2009) argues it is important to distinguish between the two different interpretations of place value – 'quantity' value aspect and 'column' value aspect. Base ten apparatus reinforces 'column' aspect and partitioning the quantity aspect. Thompson (2009) explains the difference between column value and quantity value and concludes that column value is not a necessary prerequisite for early calculation whereas an understanding of quantity value is important. Rowland et al. (2009) examine the use of resources and examples in enriching the mathematics curriculum and discuss that whatever the need, for which they are being used, the examples provided by a teacher ought, ideally, to be the outcome of a careful process of choice because some examples work better than others. Furthermore, Moyer (2001) found that teachers' statements and 'behaviours indicated that using manipulatives was little more than a diversion in classrooms where teachers were not able to represent mathematics concepts themselves' (P175). Thus the emphasis communicated was that manipulatives were fun rather than necessary to support and develop thinking.

Many lessons currently taught in English primary schools are influenced by the National Numeracy strategy (NNS, DfEE, 1999). Criticisms of the strategy include:

Nuffield year 4 project that 'found that analysis of post NNS lessons showed more opportunities to explain mental methods but little evidence of pupils discussing and evaluating different methods. One reason for this related to the objectives driven nature of lessons that allowed little room for alternatives. Aubrey (1994) found that the NNS advantages some pupils more than others, with low attainers being least advantaged. Askew and Brown (2004) noted that informed interpretation of objectives in the Framework and a move to more strategic ways of working were challenging for teachers to understand and implement. The use of objectives drives many numeracy lessons and as Denvir's (1986) research demonstrates 'I can specify what my teaching objective is before a lesson, but I cannot, in most lessons, be that much in control of the learning outcomes'.

Methodology

A number of studies (Thompson, 1984; and Beswick, 2007) have shown that case study can facilitate our knowledge and understanding of the relationship between teachers' espoused beliefs and enacted practice. In respect to this study, six primary teachers, considered locally to be effective teachers of mathematics, participated in a series of interviews and lesson observations. It was decided to work with teachers who were essentially ambassadors of the subject in order to eliminate, as far as possible, lack of confidence or enjoyment in teaching the subject. This paper reports on the findings of one of the six teachers who worked with children of age five and six years old (year 1).

An initial semi-structured interview was conducted prior to the video-taping of a series of at least three lessons, which set out to reveal colleagues' mathematical backgrounds. The aim of which was to discuss how their early experiences of school, university and, for example, family had influenced their perspectives on mathematics and its teaching. Following the initial interview each teacher was observed over a period of six to twelve weeks. Each lesson video-taped and after initial analysis where questions and issues were identified, followed by a stimulated recall interview (SRI). During this interview colleagues were asked to discuss whole class episodes of the lessons in relation to their professional decision making in respect of the chosen task and the manner in which the episode played out, e.g. the mathematical aim, the pedagogical approach used and overview. Importantly, teachers' topic choices were assumed to have been pre-determined by the statutory National Curriculum in mathematics and so SRIs focussed on colleagues' rationales in which they engaged their children during whole class episodes of their lessons. Thus, the data for each teacher comprised a pre-observation background interview and between three and six paired observations and SRIs. Data were analysed qualitatively drawing on, but not exclusively, the constant comparison exploited by grounded theorists (Strauss and Corbin, 1998).

The study and results

In English mathematics classrooms, a three-part lesson structure has become the norm since the introduction of the National Numeracy Strategy (1999). The lesson we present here was no exception. The beginning of the lesson, known as an Oral Mental Starter (OMS), began with Jane (the teacher) asking the children a series of questions to reactivate prior knowledge and introduce the short activity of practising adding on. She used flashcards with addition calculation questions e.g. 7 + 3 = ? This episode lasted nearly twelve minutes before they stopped for a 'brain break'. A brain break is a physical activity, often seen in classrooms, to clear children's minds in readiness for the next activity.

The main part of the lesson began, as most English lessons, with two lists, one of learning objectives and the other success criteria, which are read out to the children. This objectives driven approach is a procedure adopted by most teachers in England, in accordance with government guidelines (DfEE: NNS, 1999, DfES:PNS, 2003). The learning objectives state the mathematical learning of the lesson, and the success criteria list the skills needed to achieve that learning. In this lesson both lists are read out to the children. The learning objectives were:

- to partition numbers into tens and units;
- to begin to order 2-digit numbers.

These were presented in the form found in the National Numeracy Strategy Primary Framework for Mathematics: 5-6 yr olds (year one) (DfEE: NNS, 1999: 8 and 32). Although the children had started to partition numbers earlier that week, they could not remember what the word 'partition' meant. Thus Jane explained again that it meant splitting the number into tens and ones, and that that is what they would be doing later in independent work.

Jane's success criteria were:

- We will understand how to partition 2-digit numbers;
- We can show a 2-digit number by using cubes;
- We can order 2-digit numbers.

She asked the children to remind her to tick these things off at the end of the lesson. Jane had explained, in the SRI (interview) that followed the lesson, that the idea of partitioning was still new to the class and so the lesson ' *...was like one of their first steps to understanding how to partition a two digit number'*.

The first resource Jane used was illustrated on the Interactive White Board (IWB) where a set of number (place value) cards, numbers 1-9 and 10-50, like these:

$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{5}\ \boxed{7}\qquad\qquad \boxed{1\ 0}\ \boxed{2\ 0}$

They were displayed randomly at the bottom of the screen and she introduced the cards as 'place value' cards. Jane told the children that '*Some numbers have two digits in them; some have three digits in them*'. The resource was not her first choice; she explained later in the SRI that she found the cards online '*I didn't know if we had any place value cards. I would have preferred to use giant* *hands-on ones so I could show them together, take them apart etc. I would have preferred that but this was the best I could find at short notice'.*

When prompted, some of the children were able to offer examples of one, two and three digit numbers which she acknowledged and praised, however, she ignored a child who asked about four digit numbers. She then used the board to construct a two digit number, and moved a ten and a seven, placing the seven on top of the zero of the ten. She asked the children *what is the number*? she had made. Many of the children called out the number seventy. She corrected them by reiterating that it was a *seventeen* not *seventy* as it was a teen number, because '*all numbers with a one in front were teen numbers'*.

Jane explained to the class that they could split the number apart to ten and seven. She illustrated this on the board by moving the seven card away from the ten. She then reversed the process and moved the number seven back onto the zero and added the two together by counting up from ten adding seven to get seventeen.

She explained to the children that the numbers were in columns called: ones and tens. And reiterated seven ones go into the ones column and one ten goes into the tens column, just as the illustration below, stating to the children that this was '*where all two digit numbers go'*.

T	O
1	0

T	O
1	7

She repeated the process with twenty five, showing a 20 card and a five card and placed the five over the zero of the twenty. She repeats that the five is in the ones column and the two is in the tens column and partitions them as she did before. She reinforces the idea by asking the children what this means, as she makes hand gestures of pulling something apart. She repeats: *it means to split the number apart* as she moves the five away again from the twenty. Jane asks what are we left with? To which the children shout out 'twenty!'.

She then changes her approach and introduced linking cubes, as shown here, showing the children ten cubes attached to each other in a rod, and said:

> Jane: *One tower of 10 cubes here and this represents the 10s column. This is a block of 10 (*as she shows a number of cubes fixed in a rod*). It's just one block of 10.* She then held up seven orange cubes attached in a shorter length.
>
> *Jane: The orange cubes how many cubes have I got here* (showing the children the smaller length of cubes).
>
> *Class: Seven*
>
> *Jane: You think 7? Yes I have, (*and counts them). *These cubes show us the number 17 in those columns. We have one block of 10 but 7 little ones. You are going to do that soon at your tables in a minute.*

In the SRI, Jane was asked why she called the seven rod of cubes (that represented 7) the 'little ones'? She said ...*for practicality really, for showing them on the carpet. It's difficult to show them seven individual ones in your hand so ideally, that's what I would have liked them to see, and on reflection I could have perhaps stuck them with blu-tack on the little white board. ...so they could see* *them better, but I think that was the main reason it was difficult to show them seven individual ones'.* Jane then returned to the number 25 and asks:

> Jane: *how many tens are there?*
>
> Class: *Two*
>
> Jane*: yes.. we have then number 2 in our tens column, How many cubes of 10 will I need now?* She pointed to the 20 in 25.
>
> Class*: Nine... two....ten...*
>
> Jane*: It's quite difficult isn't it! With our 17 ...ok I have one in the 10s column. And one block of 10, how many do I need in this one?* (pointing to the number 7. She then points back to the number two in 25)
>
> Lisa: *we need two blocks of 10'* to which Jane praises '*we do, good girl!'*

Jane agrees and reiterated that *in the tens column we need two blocks of 10*. One child called out '*two more?'* which she rejected and repeated that they need two blocks of ten in the tens column. She again counted the two blocks in her hand and repeated again they need two blocks of tens in the tens column.

She turned to her box to get some more cubes, and a child shouted out '*we need five cubes'.* She turned with her cubes and said *that's right just five* and announced that they need *five ones for the ones column*, pointing to the ones column on the board *but two blocks of ten for our tens column. It is, quite confusing isn't it!* She stated.

The children were then told they have to have a go themselves finding cubes to represent the tens and the ones. They have to choose a number from a set of cards and represent the numbers with their cubes. She also informed them that adults in the classroom, herself and assistant, would help them to *get their heads into gear and find out what this was all about*. The children appeared to be confused at this point, but nevertheless moved to their groups to begin their task. The children had been sitting on the carpet for nearly twenty eight minutes, and appeared to be keen to move to their tables.

The seatwork lasted for nearly twelve minutes and when children appeared to struggle with making the tens and 'little ones' to partition their numbers. Jane called them back to the carpet and asked what they found difficult about the task. It was then she re-taught the idea using the linking cubes but changing the vocabulary from *blocks* of ten and *little ones* to *One group of ten or one tower of ten*. She repeated that the number one represented number ten and that it was not the number one. She explained the ones column and asked a child (Tom) to select the right number of cubes to show this number (seven). She reiterated that Tom was not getting seven *big towers* out of the box, but *just seven on its own*. She placed the two towers (ten and the seven) together to show the cubes and asked the children to help her count on from ten to seventeen. The lesson ended, the topic would not be picked up again until the following term, nearly three months later.

Discussion

The apparent frustration of all participants during the seatwork phase implied that the initial exposition had failed Jane in its objective. In the following, drawing on evidence from the SRI, we discuss the lesson and possible causes of this frustration. Firstly we look at Jane's justification of her use of resources and choice of vocabulary.

When analysing the video-tape with her, Jane reflected that the resources were not ideal. She confessed that the lesson did not go as well as she had hoped but the resources were ...*the best available to me.* She was new to the school she said ...*being new in this school I don't know.... what resources are available without hunting and taking time to have a look, 'cus obviously I haven't got the time to search through everything. So I knew we'd got cubes so ok we'd use cubes!*. She had been at the school a couple of weeks and had not looked in all the drawers within the classroom. What is interesting in this case is that Jane was a confident teacher of mathematics so perhaps believed the resource was irrelevant. But as Rowland et al argue '…whatever the purposes, for which they are being used, the examples provided by a teacher ought, ideally , to be the outcome of a careful process of choice, a deliberate and informed selection from the available options, because some are simply 'better' than others.'(Rowland et al 2009 p 72).

The aim of the lesson was to teach children to partition two digit numbers, and although she used place value cards on the IWB, she did not emphasise any connection to the order of numbers in the system, or indeed how the system is structured. This was entirely left for the children to implicitly understand, which is problematic. As Lampert (1986) and Gelman (1986) highlight the adverse affect teaching implicit understanding can have on the development of children's mathematical thinking.

The resource used initially was the IWB with number cards which she referred to as place value cards. She said ... *I found it online, I thought that would be good to use because again I didn't know if we had any place value cards. And actually I found some this afternoon (laughs) just by chance... But they were only small ones and only one set of them. So I would have preferred to use giant hands-on ones... so I could show them, You know, ... show them together take them apart etc. I would have preferred that... but this was the best I could find at short notice.*

The arrow cards, like the ones shown here, are commonly used in primary schools. The teacher would use a large dimensioned set, whereas the children would have a smaller sized set to use individually or in pairs. The PNS guidance (2003) and DfEE (NNS, 1999) suggest to teachers that they 'should use place value (arrow) cards to partition and combine numbers with zero as place holder'(p4). The resource is an abstract image rather than a more concrete one but no clear rationale is evident for this preferred choice.

It could be argued that using the number cards on the IWB instead of a teacher's set of arrow cards would have made little difference to the presentation of partitioning numbers; they simply appear on screen rather than in the teacher's hand. The representation of number still remains an abstract one. Therefore the language the teacher uses here to describe the process of partitioning is crucial in the exposition. Moreover the second learning objective in this lesson was to begin to order numbers, therefore one might expect the choice of number, in this case the number seventeen, be openly discussed and placed in order with the children. Yet its order was not referred to explicitly in this lesson or any other number, in fact no explanation or discussion about what the place value (quantity) actually means.

On the IWB Jane placed the seven over the zero of the ten. At this point several children called out *seventy* and she corrected them emphasising the ending of the word 'teen' rather than 'ty'. What she did not do was to explain why the number shown on the board was not *seventy*. She had no number line or number grid to show the children where *seventy* is located in the system, or why it sounds similar. Instead she referred to the word ending and announced that 'all numbers with a one in front were teen numbers'. This statement is more than problematic as it does not account for numbers eleven and twelve, or indeed any number between 100 – 199, or 1000-1999 and every other number the children know or might come across. The exposition demonstrated by this episode infers that Jane had limited knowledge of pedagogical concepts (PCK, Shulman, 1986). It could be argued that she had limited mathematical knowledge; however, she trained as a specialist of mathematics and achieved a very good grade in her degree. Liping Ma's (1999) view is that 'it was important for teachers to have a deep knowledge of the mathematics *at the level they were teaching* rather than having knowledge of advanced mathematics.'

Jane's progression of the lesson was draw column lines onto her board on top of the number seventeen to reiterate that the one is in the tens column and the seven was in the ones column. The children were given no explanation or reminder (prior knowledge) as to what these columns meant, or why they were important for the activity they were doing. Much of this exposition was left to the implicit learning of the children. There were clearly some children who understood the idea, as they often responded to Jane's questions with correct answers. She either ignored these answers or took them as confirmation that her explanations had been successful. After repeating the same statement, that the seven goes in the *ones* column and the *tens* go into the tens column, she qualified this fact saying 'where all the two digit numbers go'. Again her qualifying statement is problematic in this section of the exposition, as she does not expand on what she means by 'all the two digit numbers go'. What she should have explained was what all two digit numbers have in common. Instead she introduced a new number to look at, twenty five, and repeated the explanation. Askew (2011)'s argument is that

> ….the learner cannot go directly from not knowing about place value to knowing about it. Teachers provide tools and artifacts that mediate this, through which the learning is assumed to be enabled.
>
> Place value did not suddenly appear and then get applied to the real world: it is a mathematical invention that was developed as a way to organise, to mathematize the world.
>
> (Askew 2011 p.10)

Finally, Jane introduced linking cubes to the children to represent the numbers seventeen and twenty five, which is what the children's task, was to be in their seatwork. She introduced the cubes to show the partitioning of the numbers seen already abstractly on the board. The linking cubes were presented as one 'block' of ten, linked together and seven 'little ones' which were also linked in a rod, but smaller. Jane's rationale for this approach was

> *... I've used those two together before and it's been successful. Obviously not with the white board, I used the place value cards that they hold which are better.... This was good because you could see how the numbers began in their 10s and units and you could put them together and partition them again ...what I would have liked to use are the smaller place value cards at their tables, but unfortunately we didn't have the resources for that. So the next best thing was cubes... and to try to relate the partitioning and like making towers of ten to relate to the numbers.*

Jane's reflection on her strategy in the SRI revealed that she would have preferred to use arrow/place value cards (as shown above) and not the number cards and the cubes. As we have already outlined above, the children struggled with the seatwork as they could not make the connection of the number with the two rods of cubes they were asked to construct. She realised the lesson was not going to plan so stopped it and called the children back to the carpet to explain the concept again. Jane used the number seventeen again but this time used a child to join the cubes together to show the rest of the class and repeated the teaching process she went through earlier. However, she introduced new vocabulary to explain the two rods of cubes. Instead of saying here is a *block of ten and seven little ones*; she said here is a *group of ten or tower of ten and little ones.* When asked to explain this change of vocabulary in the SRI she reflected:

... if children don't understand the word 'block' then they might understand the word 'tower' or they might understand 'lots of' or they might understand other vocabulary. So I tend to throw different vocabulary at them, because some children might not understand just one of those words.

Her reflection indicated her understanding of difficulties children were having with connecting the cubes to partitioning numbers, yet she was not aware of why. Her focus of the lesson was to emphasise the learning objectives and success criteria and she spent time going through these structural elements yet did not refer at all to placing numbers in order or the quantity of number. She could provide no rationale for her teaching strategy or her use of the cubes.

Evidence now shows that these 'blocks' do not easily lend themselves to becoming tools for thinking with. People's mental models of number seem to be linked to the ordinal aspects of number: where numbers are placed in order and with respect to each other, as on a linear scale or number line (Dehaene 1999).

Why did Jane use this strategy? She stated that arrow cards were her first choice, but had none so found some place value number cards online and some cubes to illustrate the partitioning of two digit numbers. What she appeared not to have considered is the implication of using an abstract form of column value with a concrete form of number quantity. She did not provide children with the vocabulary to do this in order to make the connection between the two forms.

Conclusion

Thompson (2009) argues it is important to distinguish between the two different interpretations of place value – 'quantity' value aspect and 'column' value aspect. Base ten apparatus reinforces 'column' aspect and partitioning the quantity aspect. Thompson explains the difference between column value and quantity value and concludes that column value is not a necessary prerequisite for early calculation whereas an understanding of quantity value is important.

When analysing the national strategy documentation (DFEE,1999, 2005) the guidance to teachers informs them that abstract representations are to be favoured over more concrete materials without any clear rationale. Jane conveyed no knowledge that there could be an issue in using concrete materials (cubes) alongside the abstract form. After all, her introduction and illustration was an abstract model, she then switched to a concrete model for the children to work with. The implication of this teaching strategy was then confounded by the use of informal language when little explicit connection was made. Several issues then became apparent:

- Children's understanding of number with more than one digit in them. Do they understand where they fit into the number system?
- Children's familiarity of place value and how columns illustrate that in a two digit number where the first digit represents numbers of tens, and the second digit represents numbers of units.
- Children's familiarity of the vocabulary used in place value
- Children's familiarity with using linking cubes to illustrate number value

Jane assumed that if she showed her class the number cards to represent the two digits, and then cubes, it would help to illustrate the number symbols by offering a concrete sense of size and quantity of the number, thus the mathematical concept of place value. Research informs us however, that resources do not in themselves convey knowledge; the teacher should explicitly make the connections between the abstract and the concrete to support understanding and develop mathematical thinking. Yackel (2000), Cobb et al. (1992), and Holt (1982) state, only those who already understand the mathematical concepts being modelled will perceive the mathematics in them, as the interpretation of the individual will be constrained by prior experiences. Jane appeared not to be concerned with this, she stated that she assumed only that children were not ready for partitioning, and clearly not, that the exposition itself may have been the difficulty children were having. She changed her vocabulary indicating that she was aware of some misunderstandings in the use of the two rods of cubes. Jane was making little sense to the children. In the Nuffield key understandings in mathematics learning (2008) one of the key understandings is that children should understand: Whole numbers represent both quantities and relations between quantities, such as differences and ratio. Primary school children must establish clear connections between numbers, quantities and relations.

Jane seemed unaware of the difficulties her children experienced during seatwork, appearing to make at least three assumptions. Firstly, she assumed that her use of place value number cards facilitated her children's understanding of partitioning. Secondly, she assumed that they understood column values because she had demonstrated these on the board. Thirdly, she assumed they understood how place value columns are connected to the ordering of numbers. She asked children to use concrete materials to illustrate the quantity value of numbers, but used informal language to describe what the cubes represented. In so doing she made no explicit links between the abstract and the concrete and only compounded the confusion through the provision of further informal vocabulary. Her actions resonate closely with Cobb et al.'s (1992) finding that too often teachers assume that mathematical meaning is embodied in their representations and fail to make explicit their objectives.

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