THE INTERPLAY BETWEEN GESTURE AND SPEECH - SECOND GRADERS SOLVE MATHEMATICAL PROBLEMS

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In mathematical interactions, young learners express themselves in multiple ways to interact with each other and to get in contact with the provided culturally based mathematical environment. To deal with the complex multimodality seen in these interactions, this paper investigates the interplay between gestures and speech used by second graders while they are occupied with a geometrical problem in pairs. In the paper gesture and speech are analyzed with an interaction analysis and a detailed reconstruction of the semiotic process on a microscopic level. The main research question is: How and in what kind of modality - in gesture and/or speech - will mathematical ideas[1] be introduced, adopted, developed and/or refused by the children during their occupation with the given mathematical problem?

INTRODUCTION

During the occupation with mathematical problems in pairs, elementary school pupils gesticulate, discuss their mathematical ideas and methods, use the provided material, and possibly even write something down. In a complex kind of interplay, these diverse modes of expression do not appear sequentially, but rather simultaneously and overlap with each other. Actions can be described in speech or represented in gestures. The pupils talk about things they have written down, refer with gestures to things which were discussed before etc. In the general view the multimodal expression is generated in this way. These multiple ways of expression will be analyzed in the present paper by focusing the special relation of gesture and speech used by the learners in mathematical interactions. Looking at gestures in mathematical situations, however, is a fairly new field of research in Germany, though it is increasingly gaining international significance (cf. e.g. Arzarello & Paola 2007, Radford 2009, Sabena 2008). Gesture and speech are seen as a common language system, and display a special relationship with each other (cf. e.g. McNeill 1992, Goldin-Meadow 2003). The present paper makes use of previous approaches to multimodality as described e.g. by Radford (2009) and Sabena (2008). These approaches emphasize the significance of bodily expression in the sense of "Embodied Cognition" (Anderson 2003), and describe the body and its interaction with signs and artefacts as central sources of mathematical knowledge (cf. Sabena 2008, 19). A central role is ascribed to the body and its integration in the

[1] The term *mathematical ideas* can be understood as any kind of expressed contribution of the second graders, which contain the intention to solve the given mathematical problem.

mathematical learning environment. "[...] Thinking does not occur solely *in* the head but also *in* an *through* a sophisticated semiotic coordination of speech, body, gestures, symbols and tools." (Radford 2009, 111). A semiotic approach to the data in this paper allows a micro-analytical examination of the relationship between the gesture and speech used by second graders in mathematical interactions. A sequence out of the domain geometry will be analyzed by dint of an interactional and a semiotic perspective, and will concretize the theoretical remarks. The chosen mathematical domain of geometry underlines the significance of bodily expression, and the interplay of gesture and speech in particular.

GESTURE AND SPEECH - TWO MODES, BUT ONE SYSTEM

As elements of the semiotic repertoire which is used in interactions, gestures and speech are described as two modes of one integrated language system in the most psycholinguistic based research literature (cf. McNeill 1992 & 2005, Goldin-Meadow 2003). McNeill (1992) remarks on gestures: "They are tightly intertwined with spoken language in time, meaning, and function; so closely linked are they that we should regard the gesture and the spoken utterance as different sides of a single underlying mental process." (McNeill 1992, 1). Gestures are such stable components of the semiotic repertoire that it is probably almost impossible to suppress them for any length of time. Research shows that movements of the feet or head take over from hand and arm movements when the hands are artificially rested (cf. Goss 2010, 302). Goldin-Meadow (2003) describes this interplay between gesture and speech as having its origin in early language acquisition, where gestures acts as a facilitator and pathfinder (cf. Goldin-Meadow 2003, 17f).[2] For the research described in the present paper, one can apply the definition of gesture used by Goldin-Meadow (2003): "The criteria for a gesture thus stipulate that the hand motion (1) be produced during the communicative act of speaking [...] and (2) not be a functional act on an object or person." Thus in the present paper those gestures are focused which are produced intuitively and spontaneously during speech. My research revealed that an "act on object" and gestures cannot necessarily always be clearly distinguished from each other on the first sight. According to the multimodal paradigm, objects can be integrated into gestural argumentations without any functional action being performed upon them. In these cases, these arm and hand movements are examined in the present paper analytically as gestures. The distinction between action and gesture only emerges after a detailed analysis has been carried out (cf. Huth, in print).

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[2] The specific interplay of gesture and speech leads to the here described research focus. This does not mean that other expression modes as well as the influence of the given material on mathematical interactions are ignored or disregarded. Those aspects are considered analytically by dint of the interaction analysis, and are integrated in the interpretation of gesture, speech and their relation to each other.

In their interplay, both modes display unique characteristics in terms of their means of expression. Neither is simply a support or accessory for the other, and neither can fully replace the other. Speech can be described as a linear, hierarchically organized grammatical system that follows conventionalized rules, which are fixed within a language community. Spoken words are fleeting, but once spoken cannot be changed or taken back, but they can be further specialized. Speech can create a narrative context and can establish concepts that can outlast the present situation and be used in the future, e.g. technical terminology. On the other hand, gesture, expressed spontaneously and intuitively during speech, dose not follow any conventionalized parameters in the sense of a grammatically fixed system of rules. However, it too displays a certain fleetingness, but leaves a kind of imagistic track of movement. Deictically it can be very precise, and can refer to objects that are not currently present, even thoughts. In these cases, gestures creates quasi-real objects in the gesture space, to which can be referred to in the further interaction. Gestures which accompanying speech can experience a certain degree of conventionalization, e.g. if a gesture is repeatedly used by two speakers in the moment of interaction and established as the representation of an object (cf. Fricke 2007, 196).

Gestures in the mathematical learning process - "stepping stones" of learning

With relation to the significance of gestures in mathematics learning as an overarching research interest, mathematics education can use results of psychological and psycholinguistic studies (cf. e.g. McNeill 1992, Goldin-Meadow 2003). This chapter will introduce some studies relevant to the described research, and interpret them using perspectives of mathematics education. In relation to the learning of mathematics, Goldin-Meadow (2003) describes the theory of "matches" and "mismatches" (ibid., 25ff). Where gesture and speech express the same information, this is described as a *match*. With *mismatches*, gesture and speech convey different information that does not overlap. Goldin-Meadow (2003) was able to show that children who produced *mismatches* during the occupation with mathematical problems, were in a transition phase of learning. At first they showed *matches* with correct or incorrect strategies in speech and gesture. Then they produced *mismatches* with various correct and incorrect strategies in gesture and speech. A few time later, they showed *matches* with correct mathematical strategies. Furthermore they often used gestures to express mathematical ideas *before* they were able to explain these strategies within their speech repertoire. Thus gestures also can act as facilitators e.g. in the development of a technical language in mathematics (cf. Givry & Roth 2006). Goldin-Meadow (2003) noted from her observations that *mismatches* are an important step in the mathematical learning process (cf. ibid., 54) and also open up helpful possibilities with regard to the question of instruction (cf. ibid., 124ff). It became clear that learners in the *mismatch* phase were especially open to instructions (cf. ibid., 40ff). It needs to be noted that these results were generated by investigations with individuals in relatively clearly structured mathematical situations. According to Goldin-Meadow (2003) *mismatches* are evidently relevant

for the speaker's mental system, which means for the producer of *mismatches*. Following Goldin-Meadows description (2003) utterances can be recognized as a *mismatch* if semantically different meanings can be distinguished in the respective movements and spoken words. From an interactional theoretical perspective the question is, what interactional effects do *mismatches* have on the ongoing mathematical solving process, and how are *mismatches* perceived on an interactional level from the interlocutors? It will also be necessary to seek to describe the specific nature of *mismatches*, and to illustrate in more detail what Goldin-Meadow (2003) has already indicated with her description of a continuum of *matches* and *mismatches*. McNeill (1992) investigated the effects of *mismatches* on the listener, offering various artificially generated *mismatches* as input for the test persons (cf. ibid., 134ff). The *mismatches* were differentiated e.g. along lines of space and form.[3] McNeill (1992) was able to show, that, when the *mismatch* input they had experienced was reproduced, the test persons always tried to correct the *mismatch* in some way. The study allows to conclude that it is evidently possible to differentiate *mismatches*, and that *mismatches* have effects on the listener and the process of interpretation of utterances. The question is, if these results can be transferred to, and confirmed in relatively natural mathematical interactions of second graders, which are examined in the present paper.

Cook and Goldin-Meadow (2006) investigated the influence of gestures in mathematical instructions. They noticed that learners benefited from instructions in the learning of mathematics, at first through imitation but above al through making gestures to their own. In their study, Cook and Goldin-Meadow (2006) tried to find a reason for these results and latched onto some results form the field of behavioral research, describing, e.g. the imitation of actions as learning opportunities (cf. Carpenter, Call & Tomasello 2005). But with gestures it was not merely a question of imitating arm and hand movements. In gestures a goal is not inherent in the movement, as it is e.g. in imitating pressing on the light switch. Especially in mathematical situations of instructions learners had to understand what the showed gestures represented. Cook and Goldin-Meadow (2006) were also able to show that instructions that included gestures could have long-term effects on learning, because learners were able to transfer the gestures they learned into further mathematical tasks. McNeill (2005) describes the imitation of gestures as an insight into the mental representations of the gesture producer. According to McNeill (2005) the imitation of gestures is not only an imitation itself, but above all an insight into mental processes

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[3] *Mismatch* of space (with relation to the gesture space): In an ongoing narration, an actor is sited in a certain area of the gesture space, e.g. on the left side of the gesture space. Then the narrator uses another area as the space of reference for the same actor, which was already established for another actor. The gesture shows a shift of space whereas the speech implies continuity of reference, e.g. by using the same pronoun "he". *Mismatch* of form: A narrator uses verbs that refer to a motion but do not convey any information about the manner of this motion, e.g. *come*. In gesture then the form of motion is shown, e.g. by bouncing up and down with the hands (cf. McNeill 1992, 135).

of the interlocutor. This characterizes imitated gestures as such important events in interaction. In mathematics education Arzarello and Paola (2007) described the assumption of gestures in mathematical interactions between teachers and pupils. In a so-called "semiotic game" (Arzarello & Paola 2007, 18) the teacher integrates gestural "personal signs" of the pupils that show less technical terminology elements, into an adequate mathematical reasoning, called the "institutional signs" (ibid., 23). The integration of gestures opens up ways of learning, especially with regard to the development of an appropriate mathematical language. In these cases, the teacher is always seen as the role model for technical language in mathematics. With relation to mathematical interactions of learners, the question emerges of whether the imitation and adaption of gestures can also be reconstructed between pupils and what effects these assumptions of gestures have in the ongoing mathematical interaction. In my previous research work, I was able to show, that the former question may very much be answered in the positive, and that gestures then experienced further development in the mathematical interactions of learners. Gestural signs were taken over between the learners, they were adapted and used by the pupils for their own strategies and emerged to more developed signs. In a so-called "semiotic game among equals"[4] (Huth, in press, according to Arzarello & Paola 2007) an exchange of signs between gesture and speech could be observed: speech signs were transformed into gestural signs, and in this way they were a part of the ongoing mathematical process of negotiation. With regard to the impact of mismatches on mathematical interactions of learners, I was able to show that they were used as a kind of *pool* of mathematical strategies. These strategies then were adopted, further developed as well as integrated in own used strategies by the interlocutors, what display the importance of mismatches in these interactions (cf. Huth, in press).

Gesture and speech as linguistic signs

In order to investigate gestures and speech in mathematical utterances of learners and interpret them as linguistic signs, a theory of signs is required that enables the description of conventionalized as well as non-conventionalized signs. Peirce's concept of signs is especially appropriate (cf. Fricke 2007, 182f). Peirce places the focus squarely on the sign itself, and emphasizes the significance of the interpretation process, which is initiated when a sign is perceived as such. This aspect links in particular to an interactional theoretical perspective, and Schreiber (2010) was able to show, that Peirce's theory of signs may indeed be used to appropriately analyze mathematical interactions of learners (cf. Schreiber 2010, 56ff). Peirce describes three relata as a sign: representamen, interpretant and object (in the following often abbreviated as R, I and O). The portrayal of this concept of signs therefore involves a

[4] *Among equals* means that both interlocutors were equal concerning to their role in the mathematical interaction: No explicitly and previously defined role model of adequate mathematical reasoning is present. Both pupils participate in the interaction with their mathematical way to interpret the given problem. There is no knowing professional, and no inexperienced and unknowing novice.

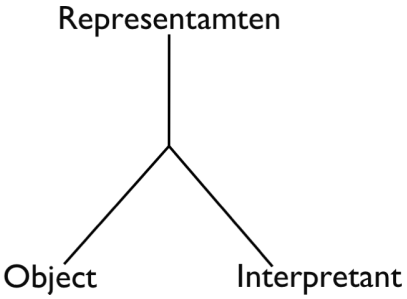

Fig. 1 The sign triad after Peirce

triad, with the help of which all three aspects of the sign can be related to each other (cf. fig.1). "A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the *ground* of the representamen." (CP 2.228).

First, the representamen pictures the external, perceivable sign and can be a word, a gesture, etc. This representamen creates in the mind of the sign reader an interpretant, which at first can be understood as the meaning of the sign for the sign reader. The object what the sign relates to, is thus also related to both the representamen and the interpretant. According to Peirce a sign only becomes a sign when it is perceived as such and interpreted by a subject. The process of signs is never-ending, since an interpretant produced in the mind of the sign reader can be expressed as a new representamen by the interpreting subject. With relation to the occupation of learners with mathematical problems, Schreiber (2010) was able to show, that the sign process is not linear, but displays a complexity, e.g. with the creation of several interpretants to one representamen; sign processes may also run parallel to each other (cf. ibid., 148ff). Schreiber (2010) developed the semiotic process cards to analyze these sign processes. In the here described research, these semiotic process cards were adopted on a multimodal level where the complexity can be confirmed with regard to different modes of expression (cf. Huth, in press). Schreiber (2010) signifies Peirce' *ground* of the representamen as "framing of the sign" (ibid., 37). According to Schreiber (2010, 36f) this framing can be described as "socially taken-as-shared and available knowledge in the sense of frames (of interpretation)" (ibid., 59). In the present Paper I will not describe these frames in detail in view of an adequate number of pages and refer to Huth (in press) concerning to further explanations.

METHOD OF ANALYSIS AND RESEARCH FOCUS

The data collection was based on the so-called "didactic design patterns" (cf. Vogel & Wippermann 2005). These patterns of description facilitate the communication of didactic knowledge, and enable it to be written up clearly in comparable categories and to be further developed. The mathematical situations[5] that were developed for the study can be assigned to three mathematical domains: geometry, combinatorics and measurement. In the paper, a sequence out of a geometrical situation will be

[5] Each situation is accompanied by an adult who presents the task and gives spare impulses if needed.

analyzed. For the qualitative data analysis, a combined method is used:[6] In a first step, transcripts[7] of the video-recorded situations are analyzed with the interaction analyses according to the interpretative research in mathematics education (cf. Krummheuer, forthcoming). In order to avoid a dominance of speech in the interpretation, the two modes - gesture and speech - are separated with each utterance, and at first only the arm and hand movements are analyzed. In the consequent procedure the alternatives for interpreting the gestures are narrowed down through the inclusion of the speech used. In this process a most probable interpretation emerges. In the second step a micro-analysis of the relationship of gesture and speech is conducted with the aid of the semiotic triad of signs after Peirce. Here the semiotic process cards from Schreiber (2010, 60ff) will be adopted and extended on a multimodal level. Following Peirce's theory of signs, two triads are used - one for gesture and one for speech - which are linked by a common interpretant. The theoretical assumptions displayed above lead to the following overarching research focus: How and in what kind of modality - in gesture and/or speech - will mathematical ideas be introduced, adopted, developed and/or refused by the children during their occupation with the given mathematical problem? Particularly interesting is what happens on the level of interaction if a *mismatch* appears. Previous results to these events in mathematical interactions of learners and to the above described "semiotic game among equals" (Huth, in press) can be used, empirically tested and possibly further developed.

THE RELATION OF GESTURE AND SPEECH IN THE EMPIRICISM

Introduction of the chosen sequence

The here described sequence is out of a video-recorded mathematical situation which can be assigned to the mathematical domain *geometry*. The situation is called *building*. Out of a given repertoire of different LEGO® DUPLO® bricks in three sizes, each of the participating pupils Jana and Ayse should at first construct a building without any demands, except to use all of the given bricks. In the following situation, each of them should emulate the building of the partner, only regarding the speech description of the interlocutor and without sight on the original building. In a mathematical sense, a three-dimensional object has to be built and described in speech. Then the design description of the interlocutor has to be used to construct a congruent three-dimensional object from the original building. Both pupils had the

[6] In the present paper the analyses will not be described in detail, but portrayed as summarized interpretations.

[7] With regard to an adequate number of pages, the transcript of the chosen and described sequence is not portrayed in the given paper. The produced utterances in speech and gesture, as well as the actions of the interlocutors can be seen in the semiotic process card (cf. fig. 5).

following LEGO DUPLO bricks at their disposal: eight 2x2 bricks[8], eight 4x2 bricks and two 6x2 bricks. Jana worked with a green colored set of these LEGO DUPLO bricks, Ayse's bricks were blue. The sets were identically concerning to the numbers of different LEGO DUPLO bricks. Ayse constructed a building (building1), which should be emulated by Jana (building2). A dividing wall between the two girls avoided, that Jana could see building1. Jana had to reconstruct the original building1 only by listening to Ayse's description in speech. This process seemed to contain some problems for the two pupils, thus, the dividing wall was put away. Jointly, Jana and Ayse now attempt to bring building2 more in line with building1. Both of them can see both buildings and also gesture can be used to explain things. Jana and Ayse visit the second grade of an elementary school with urban catchment in Frankfurt on the Main. About 70% of the visiting pupils have a migration background. Many families of the pupils have a low socioeconomic status. Jana's mother tongue is German. At the time of the video-recording Jana is 8 years old. Ayse has a Turkish background and is 7 years old. Her German proficiency is nearly on the level of mother tongue. The girl's teacher describes their mathematical knowledge at an average level. Both pupils were chosen as participants of the study because of their willingness to join in mathematical situations and according to prior consulting with their class teacher. No math tests were conducted.

Building1 (on the left) is the original building, and building2 (on the right) the replica at the beginning of the chosen sequence. The bricks are numbered, except the two 8-bricks at building 2. The red marked parts of the buildings are the subject matter of negotiation in the following interaction. The girls try to bring the red labeled part at building2 more in line with the comparable part at building1.

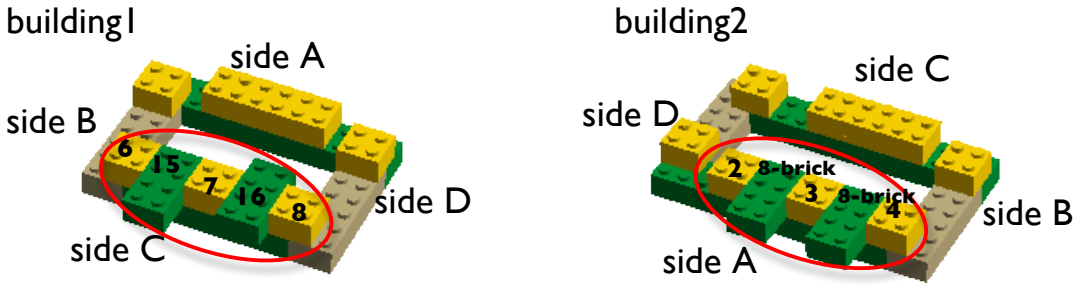

Fig.2 Building1 and building2 at the beginning of the analyzed scene[9]

At the beginning of the interaction both buildings are in front of the girls, like it is shown in the sketch below (cf. fig.3).

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[8] In the following analysis the LEGO DUPLO bricks will be signified after their numbers of knobs they have on their upper side, e.g. the 4x2 brick is called 8-brick. In the buildings most of them are numbered, e.g. brick 2, brick 15, etc.

[9] The bricks are differently colored to illustrate the transitions between them. The girls had single-colored bricks. (fig. created with LEGO Digital Designer, http://ldd.lego.com/ [27.01.2012])

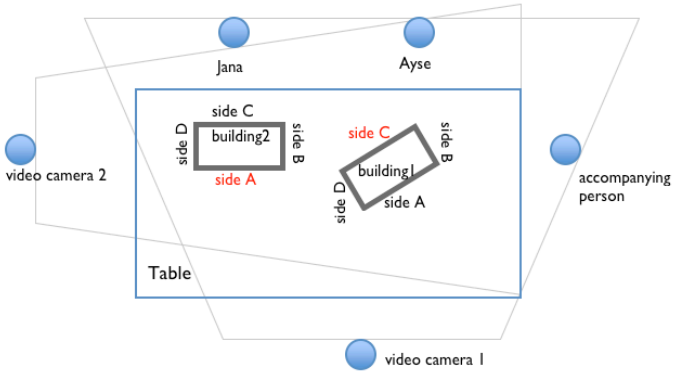

Fig.3 Sketch of the setting at the beginning of the sequence

Summarization of interpretation (interaction analysis)

At the beginning of the sequence the grasp of Ayse at the 8-brick between bricks 3 and 4 at building 2 which was set before by Jana, leads to a protest of Jana which she utters in speech. "**Ey** (inaudibly spoken) that's **right**\ I have **looked**/" is accompanied by a gesture with both flat hands. Jana's hands cover symmetrically and nearly completely side A at building 2. Thus, further manipulations on this side won't be possible any more. Even no one can take a glance at this side of the building. *Looking* seems to be the previous strategy of the two girls: They always described that they were *looking* at both buildings and then changed some bricks. Obviously Jana is sure of the correct position of the grasped brick, or the whole construction of this side of her building. Jana's wording and accentuation ("**right**") refers to her opinion that there are "right" and "wrong" positions of bricks at the building, which seems to be discovered by looking at both buildings (cf. "I have **looked**/"). The following fixed deictic gesture of Ayse on brick 15 at building1, marks on the first sight this brick as comparable to the before grasped 8-brick at building 2. The attempt to bring side A of building2 more in line with side C of building1 can be described as the main and jointly generated topic of both interlocutors after a long period of single-working before. Especially the positions of the 8-bricks at side A of building2 are the subject matter of negotiation in the following interaction. Brick 15 at building1 equates to the 8-brick which was set between brick 2 and 3 at building2. Ayse maintains the fixation by gesture of brick 15 at building1, even through the subsequent rotation of building1. The index finger marks at first the point of origin of the rotation. In speech Ayse seems to agree to Jana ("yes\ (here) and now-") and refers to "here", with which obviously brick 15 is meant. Then Ayse unfixes her index finger from brick 15 and rotates building1 several times with both hands left and right at the sides of the building. She stops the rotation when building1 is in the same adjustment like building2. This action leads to a better possibility of comparison. In speech Ayse states as opposed to her before mentioned agreement: "no but not right\ **look**\". Obviously she discovered through the rotation anything which is not right in her eyes, e.g. the position of a brick on side A of building2. Only the gesture displays the changed referent: Ayse points eight times in an energetic way on brick 16 at

building1, and obviously marks it with these gestures at first as “not right”. Only with regard to the given environment and the given mathematical problem it becomes apparent, that the comparable 8-brick at building2 is meant with “not right”, and not brick 16 itself. Furthermore, her energetic pointing seems to signalizes, that in the following a critical mathematical idea will be introduced. *Looking* is again the strategy to decide, whether a brick is set correctly or not. The comparison between both buildings becomes apparent through gestures and the strategy of looking. Gestures seem to mark not only single bricks at one building, but refers to the relation between one brick at the original building and its comparable brick at the copying building. In speech Ayse describes how the position has to be modified: “(a little) in the middle” she says. This utterance can be signified as a central mathematical idea in the following scene. While Ayse is speaking, Jana brings the 8-brick at side A of building2 between brick 2 and 3 in a new position. She sets the 8-brick “in the middle”, so that inside and outside at side A, one row of the knobs is overlapping. The position of this 8-brick, which was denoted as “right” by Jana before, is obviously no longer correct. The rotation and Ayse’s utterances of the mathematical idea to set bricks “(a little) in the middle” seems to convince Jana to change the position of this brick. One can assume, that Jana mainly considers the speech utterance of Ayse, and disregards the changed reference object by Ayse in gesture: Ayse’s previous utterances can be interpreted in the following: First, brick 15 at building1 seems to be compared to the 8-brick on building2 between brick 2 and 3. Ayse says, that this brick was set correctly. After a few moments, Ayse obviously changes her opinion and now says, that the position has to be corrected “(a little) in the middle”. This is what Jana is doing now. She didn’t recognize that Ayse changed the reference object in gesture on brick 16. In speech Ayse didn’t reveal, that another brick is meant, namely the pendant of brick 16, which is set between brick 3 and 4 at building2. Again the gesture not only marks brick 16 at building1, but brick 16 in relation to the comparable brick at building2. According to McNeill (1992, 134ff) and with regard to the level of interaction, a *mismatch* can be assumed here which is comparable to what McNeill (1992) described as a mismatch of (gesture-)space. The interpretation of Jana is based on the disregarded change of the reference object, which is not explicitly perceivable in Ayse’s speech. Only in her gesture, Ayse changed the referent to brick 16. This is the reason why Jana changed the position of the 8-brick which was set between brick 2 and 3 to the middle at side A of building2. The mathematical idea “(a little) in the middle” was introduced in speech and was set in relation to the meant bricks by gesture. But only the speech utterance was adopted by the interlocutor Jana, so that a *mismatch* on the level of interaction becomes apparent. With regard to the question what is meant with “(a little) in the middle”, and Jana’s interpretation of this utterance, Ayse does not raise a plea. Thus, this interpretation can be described as a “taken-as-shared meaning” (cf. Krummheuer 1992, 18) of the two girls: A brick is placed, so that inside and outside one row of knobs overlaps. This position is called “in the middle” by both girls. With the action of Jana, the red marked part of building2 gets a mirror image of the comparable part

of building1 with no preserving of orientation. The fictitious plane of reflection stands between both buildings (cf. fig4).

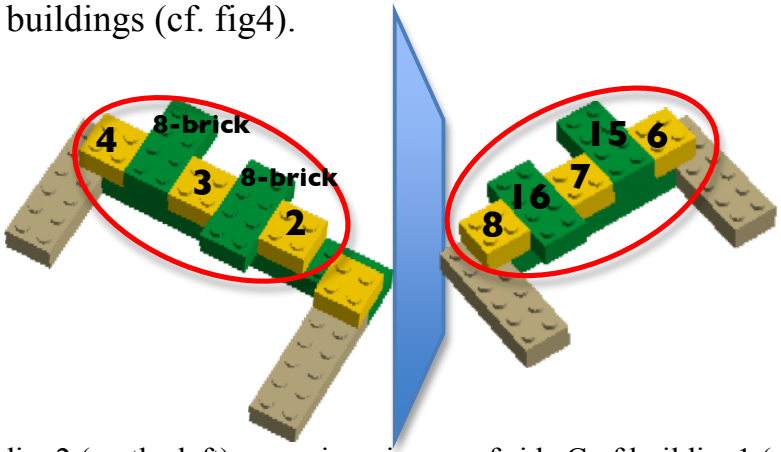

Fig.4 Side A of building2 (on the left) as a mirror image of side C of building1 (on the right)

It is still not clear, whether Jana recognizes that building2 is right-left-reversed in comparison to building1, or whether she attaches importance to this at all with regard to the solving process of the given mathematical problem. For Ayse, the fact that building2 is a mirror image of building1 obviously leads to a correction of the position of the 8-brick at building2, which was set between brick 3 and 4. Ayse obviously puts her previous remark in action: She puts the 8-brick between brick 3 and 4 at building2 "in the middle", like it is negotiated before, so that inside and outside one row of knobs overlaps. In speech Ayse explains her action and points out, which brick was meant: "This is what I've done to the middle\(inaudibly spoken) (.) like this\". Only with regard to her action it becomes apparent which brick was meant, namely the 8-brick between the bricks 3 and 4. Jana does not cover building2 again, so that Ayse is able to manipulate the 8-brick without any difficulty. The reticence of Jana here emerges possibly from Ayse's grasping of Jana's Hand. In the next utterance Jana first points at the 8-brick between brick 2 and 3 at building2 with her right forefinger. Jana lays some fingertips of her left hand down on the 8-brick between brick 3 and 4. In gesture she marks obviously the bricks which are important for her in this moment. Maybe this is an imitation and adaption of Ayse's previous behavior: Ayse also marked the important bricks by pointing several times on them. Probably Jana's fingertips at the 8-brick between brick 3 and 4 sustain these bricks for the following manipulation: Jana changes the position of the 8-brick between brick 2 and 3 at building2, so that outside two rows of knobs overlap. One can assume, that Jana recognizes Ayse's intention to create a congruent copy of building1 which has the same orientation, and that Jana now tries to support this plan. The interpretation persists, that for Jana the fact that building2 is right-left reversed to building1 is not relevant. Jana utters in speech: „No but this-(.) this was already in the middle\", what is rather contrarily to her action. The 8-brick between brick 2 and 3 was in the middle, but nevertheless Jana changes its position now. Maybe Jana does not want to concede this point to Ayse. It is also possible, that for Jana it isn't relevant whether the copy of building1 is preserving orientation or not. At the end of

the chosen sequence the adjustment of the upper row of side A at building2 to the upper row of side C at building1 can be described as completed.

Summarization of the semiotic process (semiotic analysis)

The semiotic process card, like it is portrayed below, is used as an instrument for analysis. Furthermore, it shows the process of semiotic in the described mathematical interaction of Jana and Ayse. Generally, one can see two triads[10], which are linked to each other by dint of a jointly generated interpretant. The complexity of the semiotic process (cf. Schreiber 2010) is confirmed on the multimodal level, and at points in interaction where one representamen leads to more than one interpretant (cf. triads 4a and 4b).

In the chosen sequence it is negotiated, how and which of the 8-brick at side A of building2 has to be changed according to its position, to create a replica of building1. The interaction analysis showed that an important position of these 8-bricks is called "(a little) in the middle", and means that inside and outside one row of knobs overlaps. With regard to the overarching research question, the semiotic analysis allows to observe in detail, in what kind of modality mathematical ideas will be introduced, adopted, developed and/or refused by the interlocutors. In triad1, Ayse utters as her representamen in gesture a grasping on the 8-brick at side A of building2, which was set between brick 3 and 4. It seems to be a suggestion of an action which is shown in gesture. Instead of repositioning the brick in fact, Ayse fixes the grasping. As an object one can assume the required correction of the position, shown as the suggested action. This representamen creates in the mind of Jana an interpretant that is expressed in the following triad2: In speech Jana produces „Ey (inaudibly spoken) that's right\ I have looked\". In gesture, she fully covers side A of building2 by dint of a nearly symmetrical gesture with both flat hands. With this representamen Jana obviously wants to avoid any further actions at side A of building2 which can be seen as the object of the triad. With her gesture, Jana also avoid any glance on side A. Especially with her formulation in speech in which she again refers to *looking* as an adequate strategy, this seems to be interesting. In the semiotic analysis it becomes apparent, that the speech representamen mainly includes the protest of Jana with relation to Ayse's suggested action. The gesture of Jana displays a short-term, but effective solution by dint of covering side A completely. It is evidently observable, that gesture and speech are used effectively in relation to their above described special possibilities of expression. Here, the gesture seems to be a little bit faster than speech. In the further sign process, Ayse utters her created interpretant as a new representamen in triad3, what seems to be contradictory at first sight: „Yes\(here) and now- no but not right\ **look**\". At first she obviously agree to Jana, but only a few moments later, she says „no but not right\". This discrepancy is

[10] There is a triad for gesture (on the right) and a triad for speech (on the left). When there is no speech utterance at all, there is only one triad for gesture. The triads are numbered. Parallel utterances are portrayed by parallel triads which are marked with indices a, b,... .

also getting obvious in the speech object and can only be resolved with regard to the gesture used. In the gesture object, brick 15 at building1 can be assumed, as a comparable brick to a brick at building2. Brick 15 or/and its comparable brick at building 2 are denoted as rightly sited bricks. Ayse fixed her pointing gesture at brick 15, and uses this pointing as the origin in the following rotation. Finally and by the use of further rotations, building1 is in the same adjustment as building2.

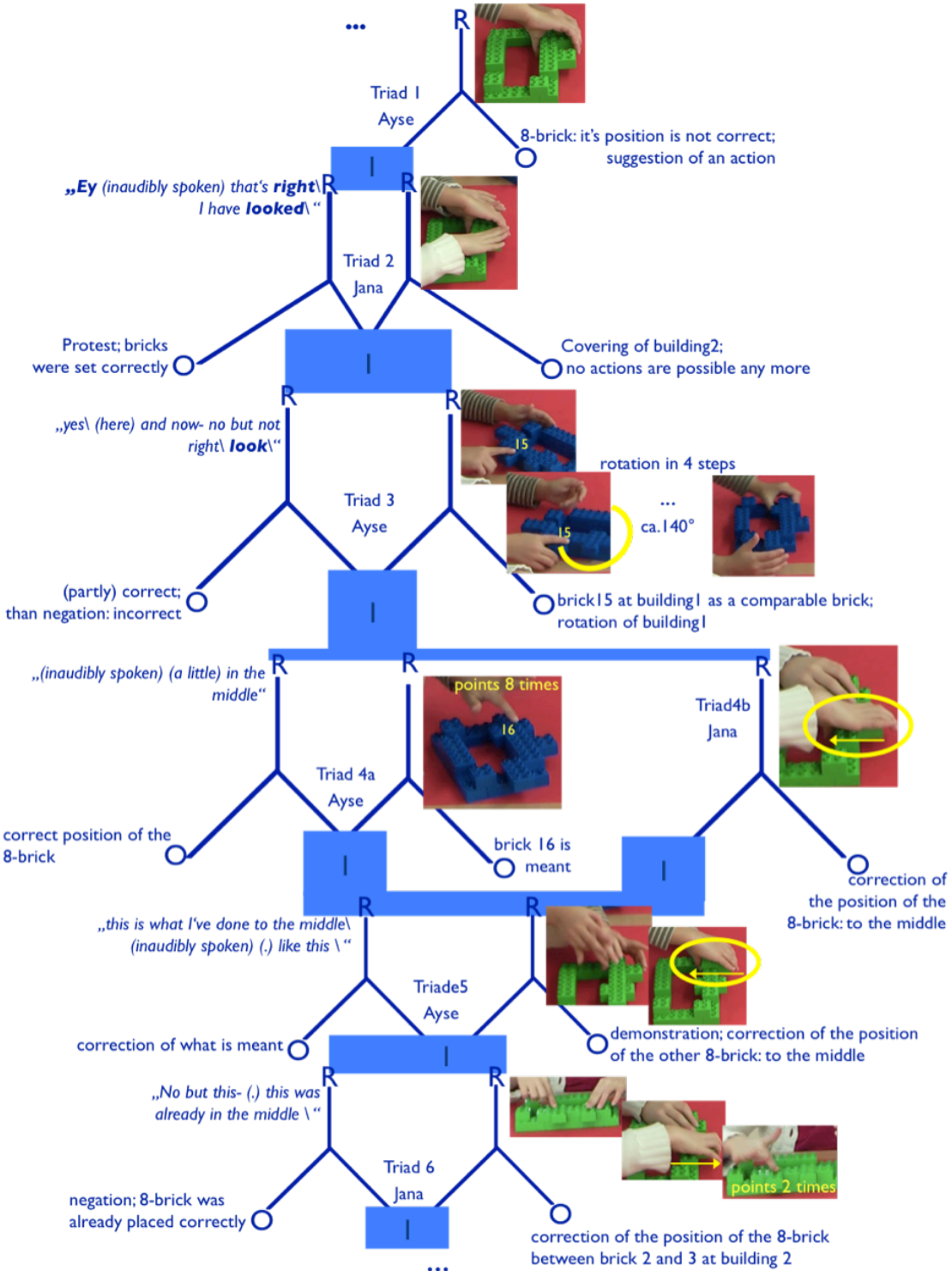

Fig.5 Semiotic process-card of the analyzed sequence

In triads 4a and 4b two interpretants are created out of the representamen of triad3. Ayse obviously carries on her reasoning in triad4a. She introduces in speech a description of the from her as correct assumed position of the 8-brick at side A of building2: “(a little) in the middle”. This can also be assumed as the speech object. In gesture the referent is changed from brick 15 at building1 to brick 16 at building1, but this changing is not explicitly expressed in speech. Both objects - in speech an in gesture - seem to refer to different meant bricks. With regard to her speech only, one can assume that again brick 15 is meant. Out of the interaction analysis it is known, that this is a critical point in the interaction of the girls, where a *mismatch* becomes apparent. By dint of the semiotic analysis it is obviously possible to document this *mismatch* in form of different objects and interpretants in the triads in relation to one representamen. At the same time in triad 4b Jana creates a new representamen, which includes an action: Jana changes the position of the 8-brick at buildig2 which was sited between brick 2 and 3. The brick now is “in the middle”. Jana disregards the change of the referent, which is only shown in Ayse’s gesture. Jana seems to attach only importance to Ayse's speech. Thus, Jana doesn’t integrate the change of referent in her interpretation, and dislocates the 8-brick “in the middle”, what wasn’t intended by Ayse. The *mismatch*, which emerges here on the level of interaction, and which is also seen in Janas Interpretation, leads to the following situation: The upper layer of side A of building2 is a mirror image of the upper layer of side C at building1. Obviously Ayse recognizes the different interpretations of both girls with regard to the 8-bricks, and the question which brick is to be set in the middle. She utters a kind of correction and tries to show which brick is meant in triad5. Her representamen in gesture refers to a kind of demonstration as the object. At first Ayse grasps Jana’s Hand, then Ayse accomplishes the repositioning on her own. Ayse sited the 8-brick between brick 3 and 4 at building2 in its “right” position with an overlapping of one row of knobs inside and outside of side A. An approximation of the object in speech and the object in gesture can be observed: Both representamens refer to a correction of the brick-position. At the end of the sequence it becomes apparent that for Jana this is no discrepancy in relation to what was done before. In speech she utters: „no but this- (.) this was in the middle already.“ With this negation she refers to the speech object which shows her opinion that the 8-brick, on which she now shows a pointing in gesture, was already set correctly before. Maybe, it is irrelevant for Jana, whether the copy of building1 is in the same orientation or laterally reversed. At the same time, she repositions the 8-brick between brick 2 and 3 at building2, so that outside two rows of knobs are overlapping. The pointing and marking of the bricks are a kind of adaption of Ayse’s previous behavior, and frames the action of Jana. She finishes her action by pointing twice at the repositioned 8-brick. Again, the gesture seems to be a bit further than the speech used what can be shown in the objects of both triads.

CONCLUSIONS

The analyses confirm the special relationship of gesture and speech. In the analyzed mathematical interaction, mathematical ideas are introduced mainly in speech, and related to the given material by gesture. Only with regard to the gestures, it is understandable which brick is meant. Furthermore, the gestures relate both buildings to each other and combine comparable bricks at the buildings. Then a pointing in form of a deictic gesture not only emphasizes one single brick, but rather this brick with regard to the comparable brick at the other building. The analyses also show, that actions and the given environment have to be considered, especially within the described theoretical framework of the multimodal paradigm (cf. Arzarello & Paola 2007). It can be evidently assumed, that the given material and the environment in total have an impact on the gestures and speech which are used by the second graders in the situation. In relation to the displayed theoretical framework of both modalities as one system, it could be further confirmed that speech and gesture have their own possibilities of expression. Second graders use both modes effectively during their occupation with a mathematical problem. In the described sequence, the mathematical idea of placing a brick “in the middle” is introduced in speech, and marks a critical point in interaction. In the following a *mismatch* (cf. the above described theory of Goldin-Meadow 2003) on the level of interaction appears and leads to different interpretations of the gesture-speech-utterance, or representamen as to say in semiotic words. In previous publications, I could reveal *mismatches*, that serve as a kind of *pool* of mathematical strategies in the interactions of second graders (cf. Huth, in print). At first sight the *mismatch* in the here described situation is comparable to this not in all respects. The *mismatch* seems to be rather on a level of *discourse* than on a level of *content* or mathematics, so to speak. The *mismatch* does not include different mathematical strategies itself to solve the given mathematical problem, but is rather on the level of the reference object, and in the following leads to mathematically interesting aspects. On the one hand, in a mathematical sense, the interlocutors rethink the positions of the critical 8-bricks at building2, and this finally leads to different solutions which can both be described as adequately in a mathematical sense. For Jana it is obviously irrelevant, whether building2 is laterally reversed in comparison with the original building1. The fact of congruency satisfies the expecting of a solution of the problem for Jana. The preserving of orientation in the congruent image of the three-dimensional object is not relevant for her. In contrast Ayse obviously attaches great importance to the preserving of orientation to create a replica of building1. On the other hand, the *mismatch* on the level of discourse is not resolved for Jana somehow: At the end of the situation, she is still of the opinion, that the bricks were already set correctly. In Jana's opinion this indeed happened before Ayse pushed for the preserving of orientation, and set the proper 8-brick “in the middle”. With regard to the mentioned *semiotic game among equals* (cf. Huth, in print) one can assume that in the situation the mathematical sign “in the middle” that is introduced in speech is transformed in actions at the different 8-bricks. The pointing gestures seem to be the instrument to

emphasize which brick is meant for both girls. And furthermore the gestures display for what brick the currently mathematical idea is significant. The gestures are adopted between the two interlocutors to underline their own mathematical opinion about what for them can be regarded as an image of building1 according to the expected mathematical solution of the problem. By gesture, the girls refer not only to single bricks, but rather on the relevance of the bricks concerning to the given mathematical idea to put them "in the middle", and they refer to bricks and their comparable brick at the other building.

In the future research work, these results have to be confirmed in further examples of data and by dint of the displayed analytic instruments. Theoretically the *semiotic game among equals* has to be described in detail, and especially in relation to the roles of *mismatches* in mathematical interactions of learners. These theoretical descriptions can be used in the future to differentiate the *mismatch* theory on the level of mathematical interaction, as well as concerning to the question of different *mismatches*, e.g. on the level of discourse and mathematics. Possibly further levels can be found with regard to the question how *mathematically* a mismatch can be.

References

Anderson, M. L. (2003). Embodied Cognition: A field guide. In: Artificial Intelligence 149, 91-130.

Arzarello, F. & Paola, D. (2007). Semiotic Games: The Role of the Teacher. In J. H. Woo, H. C. Lew, K. S. Park and D. Y. Soe (Ed.). Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, 2, 17-24, Seoul: PME.

Carpenter, M., Call, J., & Tomasello, M. (2005). Twelve- and 18-month-olds copy actions in terms of goals. In: Developmental Science, 8(1), 13–20.

Cook, S. & Goldin-Meadow, S. (2006). The Role of Gesture in Learning: Do Children Use Their Hands to Change Their Minds? In: Journal of Cognition and Development, 7(2), 211-232.

Fricke, E. (2007). Origo, Geste und Raum. Lokaldeixis im Deutschen". Berlin: de Gruyter.

Goldin-Meadow, S. (2003). Hearing Gesture. How our Hands help us Think. Cambridge, Belknapp Press of Harvard University Press.

Goss, J. (2010). Free your feet – Free your mind: An analysis of spontaneous coverbal foot gestures. Talk at the 4th conference of the international society of gesture studies ISGS. Gesture – Evolution, Brain and Linguistic Structures, July 2010, Frankfurt on the Oder, 302.

Givry, D. & Roth, W.-M. (2006): Toward a New Conception of Conceptions. Interplay of Talk, Gestures, and Structures in the Setting. In: Journal of Research in Science Teaching, 43 (10), 1086–1109.

Huth, M. (in print). Das Zusammenspiel von Gestik und Lautsprache in mathematischen Gesprächen von Kindern. In: B. Brandt, G. Krummheuer and R. Vogel (Ed.). Die Projekte erStMaL und MaKreKi, Mathematikdidaktische Forschung am “Centre for Individual Development and Adaptive Education” (IDeA), 1, Münster: Waxmann.

Krummheuer, G. (forthcoming). Die Interaktionsanalyse. In: F. Heinzel (Ed), Methoden der Kindheitsforschung, 2, Weinheim: Juventa.

Krummheuer, G. (1992): Lernen mit "Format". Elemente einer interaktionistischen Lerntheorie. Diskutiert an Beispielen mathematischen Unterrichts. Weinheim: Deutscher Studien Verlag.

McNeill, D. (2005). Gesture and Thought. Chicago, London: University of Chicago Press.

McNeill, D. (1992). Hand and Mind. What Gestures Reveal about Thought. Chicago, London: University of Chicago Press.

Peirce, C. S. Collected Papers of Charles Sanders Peirce Volumes I-VI. Hartshorne, C. & Weiss, P. (Ed) (1991-1935). Cambridge, Mass: Harvard University Press.

Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings”. In: Educational Studies in Mathematics,70 (3), 111-126.

Schreiber, C. (2010). Semiotische Prozess-Karten – Chatbasierte Inskriptionen in mathematischen Problemlöseprozessen. Empirische Studien zur Didaktik der Mathematik, 1, Münster: Waxmann.

Sabena, C. (2008). On the Semiotics of Gestures. In: L. Radford, G. Schubring and F. Seeger (Ed.), Semiotics in Mathematics Education, Rotterdam, Sense Publishers, 19-38.

Vogel, R. & Wippermann, S. (2005). Transferstrategien im Projekt VIB – didaktische Design Patterns zur Dokumentation der Projektergebnisse. In: Ch. Bescherer (Hrsg.), Einfluss der neuen Medien auf die Fachdidaktiken. Baltmannsweiler: Schneider, 39-60.

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