

# **Developing Critical Thinking Skill in Mathematics Education**

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*In light of the importance of developing critical thinking, and given the scarcity of research on critical thinking in mathematics education in the broader context of higher-order thinking skills, we have carried out a research that examined how teaching strategies oriented towards developing higher-order thinking skills influenced the students' critical thinking abilities. The guiding rationale of the work was that such teaching can foster the students' skills of and dispositions towards critical thinking. In this research, a primary attempt has been made to examine the relations between education for critical thinking and mathematics education through examining teaching and learning critical thinking according to the infusion approach, which combines critical thinking and mathematical content, in this case, "Probability in Daily Life." The purpose of this research was to examine how and to what extent it is possible to develop critical thinking by means of the learning unit "Probability in Daily Life" using the infusion approach. The research questions were: (i) To what extent does the study of "Probability in Daily Life" in the infusion approach contribute to the development of critical thinking dispositions and abilities? (ii) What are the processes of construction of critical thinking skills during the study of the "Probability in Daily Life" learning unit in the infusion approach? This study showed that it is possible to incorporate into regular schools activities that will develop the students' critical thinking and mathematics creativity. The subject matter was part of the high-school curriculum; therefore it does not take time away from the regular syllabus.*

## **Theoretical Background**

In the field of education, it is generally agreed upon that Critical Thinking capabilities are crucial to one's success in the modern world, where making rational decisions is increasingly becoming a part of everyday life. Students must learn to test reliability, raise doubts, investigate situations and alternatives, both in school and in everyday life. As will be discussed, as well as acquiring CT, it is important to assess students'

application of their CT in different contexts. Many studies investigate CT in general, or in fields other than Mathematics, but few discuss CT in Mathematics. This study will explore CT in the context of a probability session. This research is based on three key elements: (a) Ennis' CT taxonomy that includes CT skills ( Ennis, 1989), (b)The Learning unit "probability in daily life" (Lieberman & Tversky 2002). (c) The Infusion approach between subject matter and thinking skills (Swartz, 1992).

### **Critical Thinking skills by Ennis (1989)**

Ennis defines CT as “reasonable reflective thinking focused on deciding what to believe or do.” In light of this definition, he develops a CT taxonomy that relates to skills that includes not only the intellectual aspect but the behavioral aspect as well. In addition, Ennis's taxonomy includes skills, dispositions and abilities (1989). The details of this alignment follow: *Dispositions towards CT* – A defined search for a thesis, questions and explanations, being sufficiently informed, using reliable sources, taking the overall situation into account, being relevant to the main issue, looking for alternatives, seriously considering other peoples' point of view, the suspension of judgment, taking a stand, striving for accuracy, dealing with the components of an issue in an orderly fashion, and sensitivity. *Abilities in CT* – focusing on the question, analyzing arguments, raising questions, evaluating the source's reliability, deduction, induction, value judgments, concept definition, assumption identification, taking actions, and interacting with others. Ennis claims that CT is a reflective (by critically thinking, one's own thinking activity is examined) and practical activity aiming for a moderate action or belief. There are five key concepts and characteristics defining CT according to Ennis: *practical, reflective, moderate, belief* and *action*.

### **Learning unit "probability in the daily life" (Lieberman and Tversky 2002)**

In this learning unit, which is a part of the formal syllabus of the Ministry of Education, the student is required to analyze problems, raise questions and think critically about the data and the information. The purpose of the learning unit is not to be satisfied with a numerical answer but to examine the data and its validity. In cases where there is no single numerical

answer, the students are required to know what questions to ask and how to analyze the problem qualitatively, not only quantitatively. Along with being provided with statistical instruments, students are redirected to their intuitive mechanisms to help them estimate probabilities in daily life. Simultaneously, students examine the logical premises of these intuitions, along with misjudgments of their application. Here, the key concepts are: probability rules, conditional probability and Baye's theorem, statistical relation, causal relation and judgment by representative.

### **The Infusion approach (Swartz, 1992)**

There are two main approaches to fostering CT: the *general skills* approach which is characterized by designing special courses for instructing CT skills, and the *infusion approach* which is characterized by providing these skills through teaching the set learning material. According to Swartz, the Infusion approach aims for specific instruction of special CT skills during the course of different subjects. *According to this approach there is a need to reprocess the set material in order to combine it with thinking skills.*

This report is a description of an initial study, a snap shot that focused on one session and demonstrates the entire study. In this report, we will show how the mathematical content of "probability in daily life" was combined with CT skills from Ennis' taxonomy, reprocessed the curriculum, tested different learning units and evaluated the subjects' CT skills. Moreover, one of the overall research purposes is to examine the effect of the Infusion approach on the development of critical thinking skills through probability sessions. The comprehensive research purpose will be to examine the effect of learning by the Infusion approach using the Cornell questioners (a quantitative test) and quantitative means.

### **Methodology**

In this article we ask how can CT skills be incorporated into a structured Mathematics session, such as a probability session?

Fifty five children between the ages of fifteen and sixteen participated in extra curriculum program aimed to enhance students from different cultural backgrounds and socio-economical levels. An instructional experiment was conducted in which probability sessions were combined

with CT skills. The experiment constituted fifteen sessions of 90 minutes each, during the course of an academic year, in which the teacher was also one of the researchers. Data collection was conducted by way of triangulation: (1) Personal interviews – conducted randomly. Five students were interviewed at the end of a session and one week after. The personal interviews were conducted in order to reveal a change in the students' attitudes throughout the academic year (2) The students' products were collected: exams, in-class papers and homework (3) All sessions were documented and analyzed – the sessions were recorded and transcribed. The teacher kept a journal (log) on every session. Data was processed by means of qualitative methods which enabled to follow the students' patterns of thinking and interpretation with regards to the learned material in different contexts.

### **The teaching experiment**

A probability unit comprised of fifteen sessions of ninety minutes each was taught. The probability unit combines CT skills with the mathematical content of "probability in daily life". This new probability unit is a processed unit that includes questions taken from daily life situations, newspapers and surveys, and combines CT skills. Each of the fifteen sessions that comprise the probability unit has a fixed structure – a. A generic (general) question written on the blackboard; b. The student's reference to the question and a discussion over the question using probability and statistical instruments and; c. An open discussion that combines practicing the CT skills. Table 1 depicts an example for a session. The mathematical subjects learned during these fifteen sessions were: Introduction to set theory, probability rules, building a 3D table, conditional probability and Baye's theorem, statistical correlation and causal correlation, Simpson's paradox, and judgment by representative.

The following CT skills were incorporated in all fifteen sessions: A clear search for a thesis or question, the evaluation of reliable sources, identifying variables, "thinking out of the box," and a search for alternatives. The following is a description of a session discussing the subject of statistical relation and causal relation, which involves CT skills already practiced in past sessions, and new CT skills.

### **Closing remarks**

The research described here constitutes a small step in the direction of developing additional learning units within the traditional curriculum. Current research is exploring additional means of CT evaluation, including: the Cornell CT scale (Ennis, 1987), questionnaires of varied approaches, and a comprehensive test composed for future research.

The general educational implications derived from this research can and should be used to lever the intellectual development of the student beyond the technical content of the course, by creating learning environments which foster CT, which will, in turn, encourage him to inquire the issue at hand, evaluate the information and react to it as a critical thinker. It is important to note, that in addition to the skills mentioned above, in the course of this session the students also gain intellectual skills such as conceptual thinking and class culture that (climate) foster CT. Students practice critical thinking by probability, while the presented article constitutes the basis for practicing critical thinking skills together with the subject of probability. In this session, the following skills are practiced: referring to information sources (paragraph 2), encouraging open-mindedness and mental flexibility (all questions), a change in attitude (paragraph 28) and searching for alternatives (paragraph 31). A very important intellectual skill is the fostering of cognitive determination –to be able to express one's attitude and present an opinion that is supported by facts (paragraph 17-20). In this session, students are shown to be searching for the truth, they are open minded and are self confident. In other words, they practice critical thinking skills.

## **References**

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