Dynamic mathematics and computer-assisted testing: GeoGebra inside TAO

Yves Kreis

Research Unit Educational Measurement and Applied Cognitive Science
University of Luxembourg
Faculty of Humanities, Arts and Educational Science
B.P. 2
L-7201 Walferdange
Luxembourg

+352 466644 9329 • +352 466644 9453

yves.kreis@uni.lu • http://www.emacs.uni.lu/

Carole Dording

Research Unit Educational Measurement and Applied Cognitive Science
University of Luxembourg
Faculty of Humanities, Arts and Educational Science
B.P. 2
L-7201 Walferdange
Luxembourg

carole.dording@uni.lu • http://www.emacs.uni.lu/

Vincent Porro

Département Centre d'Innovation par les Technologies de l'Information
Centre de Recherche Public Henri Tudor
29 avenue John F. Kennedy
L-1855 Luxembourg-Kirchberg

vincent.porro@tudor.lu • http://www.citi.tudor.lu/

Raynald Jadoul

Département Centre d'Innovation par les Technologies de l'Information
Centre de Recherche Public Henri Tudor
29 avenue John F. Kennedy
L-1855 Luxembourg-Kirchberg

raynald.jadoul@tudor.lu • http://www.citi.tudor.lu/

Abstract

In this article we present the integration of the dynamic mathematics system GeoGebra into the computer-assisted testing framework TAO. One chapter is dedicated to the explanations of the tools involved as well as their communication. Another one focuses on a small-scale test, explaining the building of such a test, describing its realization as well as the constraints met and reporting on the collected data as well as its possible usage.

Keywords

Dynamic Mathematics, Computer-Assisted Testing, GeoGebra, TAO, Interactive Geometry, Dynamic Geometry

1 Introduction

Advances in educational practices, either in learning or assessment, along with technology progress can offer more opportunities to transpose powerful classical educational tools that have proved to be efficient in "real-life" of the classrooms to their "virtual" counterparts. In this perspective, many attempts have been made to transpose to computer versions, geometry and algebra exercises, traditionally attached to the blackboard or the paper notebook.

However, regarding these attempts, in order to address the various application contexts with respect to the different purposes and pedagogical goals, the process requires the support of very flexible tools that can be easily adapted to the learning and assessment requirements; this is a major prerequisite for the adoption of the technology in the domain of education.

2 TAO

First we emphasize the strength of the dynamic mathematics system GeoGebra, then we describe the TAO platform oriented to the technology-based assessment domain and finally we explain how TAO smoothly supports powerful pedagogical tools like GeoGebra.

2.1 GeoGebra

The drag mode in a Dynamic Geometry System (DGS) allows an interactive exploration of the represented mathematical situation and a dynamic transformation of the displayed mathematical objects far beyond the possibilities of paper-and-pencil geometry (Laborde 2002; Straesser 2002; Kreis 2004). Besides the need for a combination of DGS and Computer Algebra System (CAS) has been regularly brought up for discussion (Schumann 1991; Schumann and Green, 2000; Oldenburg 2005; Hohenwarter and Fuchs, 2005) and lead to the development of GeoGebra by Markus Hohenwarter starting in 2001.

GeoGebra (2009) is characterized by the bidirectional link between the graphical representation (point, segment, square, line, etc.) and the corresponding numeric or algebraic object (coordinates, length, area, equation, etc.). Moreover it accepts functions and boolean expressions as input, contains a spreadsheet view and will soon provide access to the CAS through another view.

The possibilities for mathematics teaching offered by GeoGebra exceed the ones of DGS and CAS. Haftendorn (2005) emphasizes this evolution and speaks of dynamic mathematics, while Sangwin (2007) stresses the unprecedented opportunities of the Dynamic Mathematics System (DMS) GeoGebra for mathematics education.

2.2 Computer-assisted testing

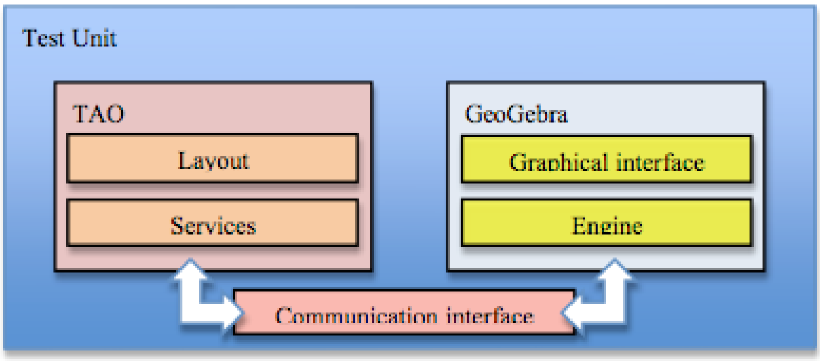
With the advent and extension of the information and communication technologies (ICT), general principles of traditional paper-and-pencil educational assessment – cognition, observation, and interpretation (Pellegrino et al. 2001) – have evolved to integrate additional technical dimensions and constraints, such as, accessibility, security, and quality of measurement instruments.

Even though the complexity of these new dimensions is not trivial, AL-Smadi and Gütl (2008) recently stressed the increasing need for technology-based assessment for modern environments in educational settings. The TAO platform is one of the candidates addressing this need (Plichart et al. 2004; Martin et al. 2005).

2.3 TAO

TAO is a sustainable generic open-source platform supporting the different phases of technology-based assessment (TBA). This cutting-edge tool has resulted from the collaboration of the University of Luxemburg and the CRP Henri Tudor (Latour and Martin 2007).

For the past seven years, it has gained the necessary flexibility to cope with the requirements imposed by various contexts extending from the simplest cases (e.g. one teacher assessing the acquisition of a specific concept by a class) to the largest and extremely complex cases (e.g. the computer-based cognitive assessments of the OECD programs). Nevertheless, even though the current system proved itself successful in a wide range of application domains, the time has come to push further the limits of the platform. Thus, building up on its intrinsic multimedia capabilities and its recent enrichment with new “semantic oriented” services that will be the foundation of the next evolution of the Web approach (Web 3.0), the platform has opened new directions of research in formative assessment and has secured partnership by extensive integration with powerful third-party solutions. Regarding this new opportunities and bearing the will to go beyond the barriers of closed question formats, the TAO platform aims to integrate the capabilities to evaluate automatically different types of open questions. The TAO-GeoGebra project subscribes to this effort. The TAO platform supports GeoGebra as a highly interactive stimulus. All interactions of a student using GeoGebra are recorded. These records are structured in the form of “events”. During a test, dealing with a test item, a student will thus generate huge amounts of events that will be stored into the TAO platform as the “history” of the user’s activities. GeoGebra is a full-featured application and it is able to provide precise endorsements based on the actions operated by the tested. However, GeoGebra is not built to manage a complete sequence of test units and items composing a test. Neither does it manage the various dimensions of the test (e.g. the student authentication and test assignment) nor does it perform complex analyses based on patterns recognition on the flow of the generated events. That is where the TAO platform comes and complements the power of GeoGebra.



To permit and ease this collaboration, the TAO platform has provided a richer synchronous communication interface where the services of TAO (events logging, context restoration, …) are exposed to the GeoGebra environment and GeoGebra stimulus offers an well-defined access to its internal engine.

3 Testing

As a start we explain TAO tests containing GeoGebra items. Then we describe its realization in classes together with the encountered limitations and problems. Finally we explain the collected data and illustrate its exploration.

3.1 GeoGebra inside TAO

A TAO test is defined in an XML file containing the different items, which can be of different types (MCQ, Open Answer, GeoGebra, …), their sequence and their weight for the final result. Such a test can be available in different languages letting the tested switch languages during testing.

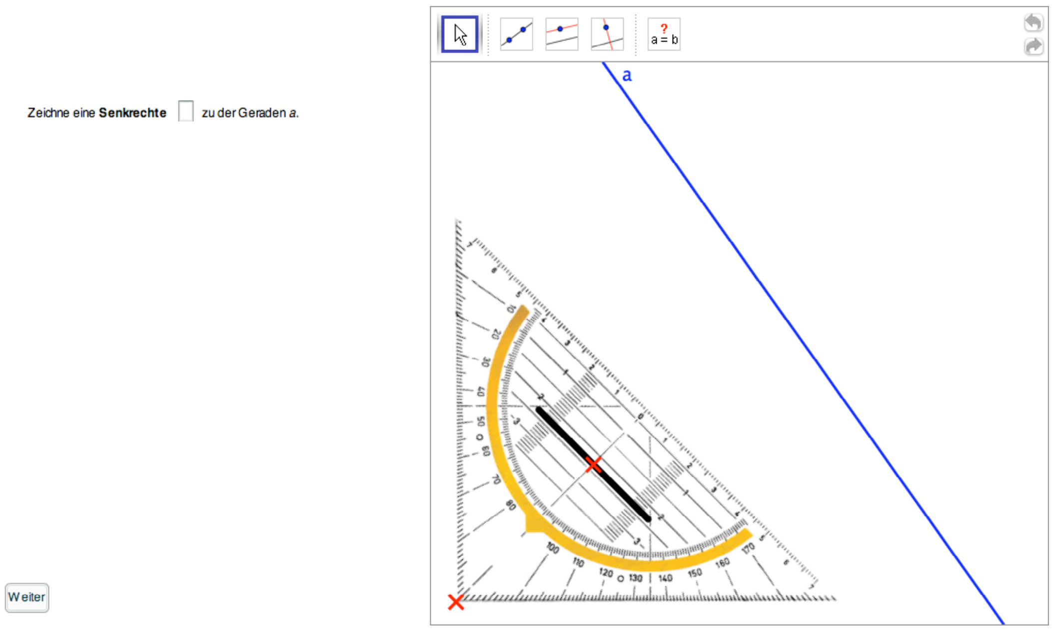


Figure 1: Draw a perpendicular line to the line *a*

An item of type GeoGebra is also defined in an XML file containing the stimulus – the GeoGebra file loaded when the item is first displayed –, the presentation and one or more inquiries with their corresponding endorsements. An inquiry can contain a field to collect the name of a GeoGebra object (compare figure 1) that may be used to build the endorsement ([toreplace] $\perp a$) evaluated by GeoGebra.

The option that an item may consist of multiple inquiries offers the chance to start with a stimulus and request from the tested to create stepwise a final configuration, which can be questioned again (compare figure 2). Each inquiry is evaluated separately and the endorsement of the item is calculated based on the specified evaluation rule.

3.2 Conducting the test

Generally there are three options to conduct a TBA-test:

1. use a bootable CD-ROM with a complete operating system (and the test)
2. use a portable computer lab with a suitable preconfigured environment
3. use the locally available computers

All three options have their pros and contras. Booting from CD-ROM is not allowed everywhere while it is an easy method to have all necessary applications available in the correct version. Using a portable computer lab has the same advantages with even less efforts but is only an option for locally conducted surveys while convincing the network administrator to gain access to his intranet might be a hard piece of work. Access to the locally available computers is normally granted without trouble but the availability of the necessary applications or the correct versions of them might be a problem.

During the small-scale study – more details in the next section – options 2 and 3 have been tested and resulted in comparable results; the organizational efforts – getting access to the local network vs. organizing availability of the correct applications – were analogous.

a) Zeichne eine **Parallele** ☐ zu der Geraden *h* durch den Punkt.

a) Draw a parallel line …

b) Zeichne eine **Senkrechte** ☐ zu der Geraden *h* durch den Punkt.

b) Draw a perpendicular line …

c) Kreuze die richtigen Antworten an:

Die schwarzen Geraden sind ☐ senkrecht ☐ nicht senkrecht ☐ parallel ☐ nicht parallel zueinander.

c) Mark the correct answers

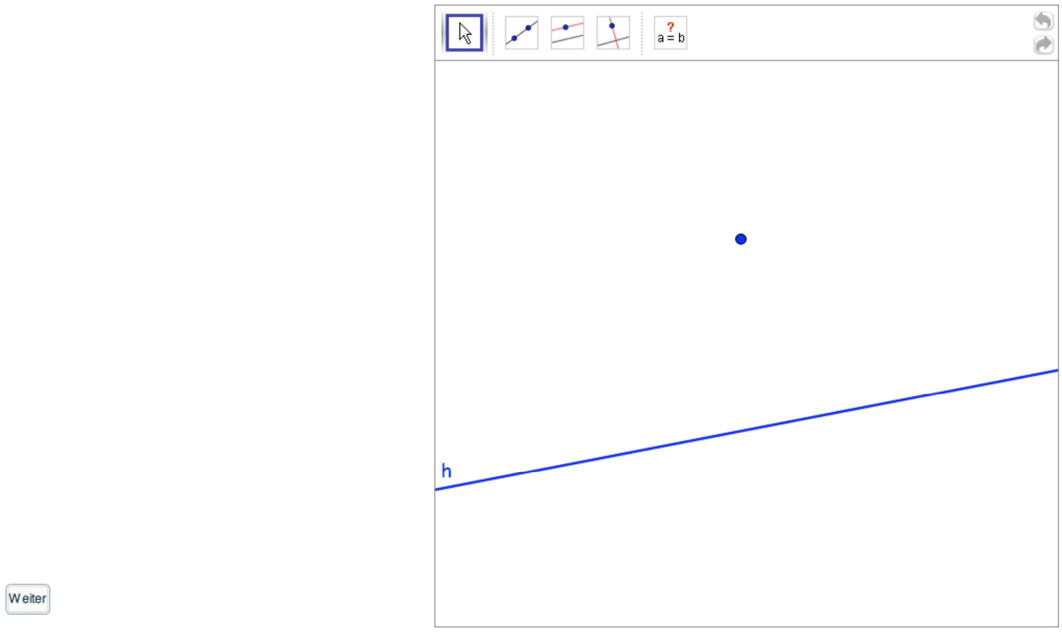


Figure 2: Multiple inquiries may form an item

However three major problems were encountered during field-testing:

1. memory limitations of Flash requiring action from the tested
2. establishment of link between GeoGebra and TAO requires time
3. missing back button, especially for multiple inquiries in an item

The first problem involves the local storage settings of Flash, which are by default 1 MB. If the tested does not accept the increase in storage then parts of the trace might be lost. This drawback can however be solved by using a dummy first test item requesting a lot of local storage and repeating until accepted by the tested.

The second difficulty mainly requires patience from the tested. It usually occurs on older computers requiring more time to start up and initializing Flash and Java. It can be partially solved in the same manner the first one is, e.g. by including a dummy first test item requiring, thus launching and initializing, Flash and Java.

The third one has mostly been noticed on items with multiple inquiries (compare figure 2). The children have used the “Weiter” (Next) button to continue but they didn’t see that the inquiry changed leaving the stimulus alone. Thus they used the “Weiter” button once again leaving a sub-question unanswered and resulting in the fact that the next one could not be answered anymore. The only solution to overcome this dilemma – apart of making the children aware of the behavior – is to add a “Zurück” (Back) button to the items. This requires however to restore the previous answers and – if the step back occurs towards the previous item – to reinstate the context of the stimulus.

3.3 Collected data

The collected data is part of a small-scale project (Kreis & Dording 2009) at the University of Luxembourg: 59[1] children of 9 years have participated in the experiment; 30 of them followed a traditional paper-and-pencil geometry course while the other 29 worked additionally with GeoGebra.

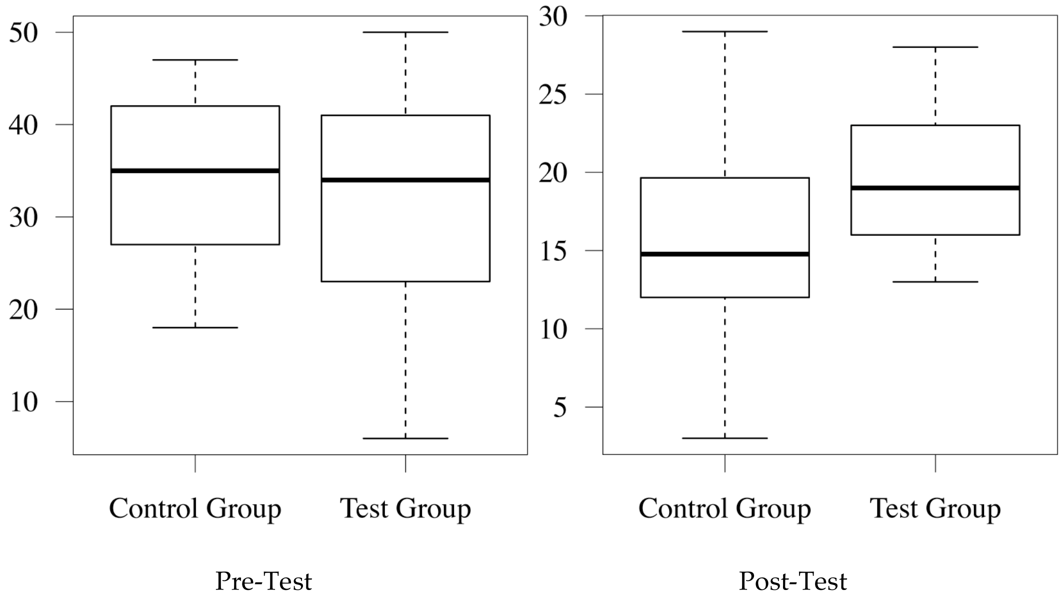
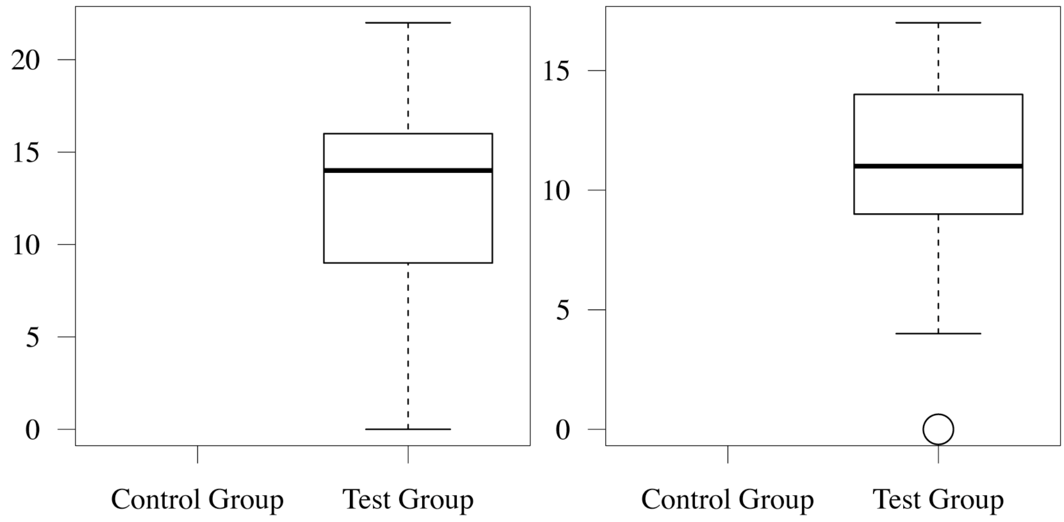


Figure 3: Box plots for the pre-test and the post-test

Kreis et. al (2010) have shown that (compare figure 3), although both groups cannot be distinguished significantly (t[57] = -0,44; p = 0,66; d = -0,12) regarding the results of the pre-test (maximum 50), both groups differ significantly (t[57] = 3,16; p = 0,002) and content wise with a large effect (d = 0,84) – based on the suggested values by Cohen (1988) – for the post-test (maximum 31).

[1] Initially 200 children were planed; unfortunately some classes left for different reasons just before or even during the experiment.



with multiple inquiries (#23) without multiple inquires (#17)

Figure 4: Box plots for the TBA-test using GeoGebra inside TAO

The results of the TBA-test (maximum 23) compared to the post-test – for the test group – can be considered comparable especially if we take into account the problem with multiple inquiries in an item described in the previous section (maximum 17). Thus the test group has not only performed well in the paper-and-pencil test but they also achieved considerable results in the computer-based test.

Nevertheless there is further information contained in the collected data. A snapshot of the stimulus is stored before stepping towards the next item. It contains valuable and easily exploitable data. Regarding for example the questions mentioned in figure 2, a parallel or perpendicular line could have been drawn although it is not passing through the point as requested. Or the opposite could be true: the tested has drawn a line passing through the point that is neither parallel nor perpendicular to the existing line as required. Or maybe the terms parallel and perpendicular have "only" been interchanged. All these possibilities influence which answers are correct or not for question c and thus need to be considered for an accurate evaluation.

Besides a detailed history with all recorded events generated by the tested is available. As it is time stamped it allows creating a realistic film representing most of the actions done by the tested on the screen. This movie can be used – combined with videotapes or not – to analyze the exact behavior of the child solving the exercise.

4 Conclusions

In this article, we presented GeoGebra, a powerful dynamic mathematics system, as well as the TAO platform that aims to provide the flexibility various kinds of tools and multimedia interactive elements to rise to the challenges of a modern education. Furthermore we have demonstrated that children used to technology performed in our small-scale study at least as well in technology-enhanced assessment as they did using paper-and-pencil.

The various and precious information stored by the TAO system are generally underestimated. They can also be used during class and not only in a final test at the end of the learning process. Thus technology-enhanced assessment – including formative assessment – can facilitate teaching, learning, and testing in ways that paper-and-pencil cannot (Bennett, 2002).

Acknowledgments

The integration of GeoGebra into TAO was funded by the University of Luxembourg through the research project GeoGebraPrim (research grant F3R-EMA-PUL-07GGEP). We wish to thank the participating children and their teachers for their efforts, their patience and their time.

References

AL-Smadi, M., & Gütl, C. (2008). Past, present and future of e-assessment: Towards a flexible e-assessment system. Presented at the International Computer Aided Learning, Villach, Austria.

Bennett, R. E. (2002). Inexorable and Inevitable: The Continuing Story of Technology and Assessment. The Journal of Technology, Learning, and Assessment, *1*(1).

Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences* (Second Edition). Hillsdale: Lawrence Erlbaum Associates.

Haftendorn, D. (2005). Dynamische Geometrie - Bewegung beflügelt Verstehen. In G. Graumann (Ed.), *Beiträge zum Mathematikunterricht 2005* (pp. 235-238).

Hohenwarter, M., Borcherds, M., & Kreis, Y. (2009). *GeoGebra 3.2*.

Hohenwarter, M., & Fuchs, K. (2005). Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. In C. Sárvári (Ed.), *Computer algebra systems and dynamic geometry systems in mathematics teaching* (pp. 128-133).

Kreis, Y. (2004). *Mathé mat TIC : Intégration de l'outil informatique dans le cours de mathématiques de la classe de 4e* (Travail de candidature). Ministère de l'Éducation nationale et de la Formation professionnelle, Luxembourg.

Kreis, Y., & Dording, C. (2009). GeoGebraPrim – GeoGebra for Primary School. Presented at the The Ninth International Conference on Technology in Mathematics Teaching (ICTMT 9), Metz, France. Retrieved from http://publications.uni.lu/record/4580/files/ICTMT9-GeoGebraPrim.pdf

Kreis, Y., Dording, C., & Keller, U. (2010). GeoGebraPrim – GeoGebra in der Grundschule. In *Beiträge zum Mathematikunterricht 2010*, Beiträge zum Mathematikunterricht. Presented at the Gemeinsame Jahrestagung der DMV und GDM, München, Deutschland: WTM-Verlag. Retrieved from http://publications.uni.lu/record/4584/files/BzMU10_GeoGebraPrim.pdf

Laborde, C. (2002). Integration of technology in the design of geometry tasks with Cabri-géomètre. *International Journal of Computers for Mathematical Learning*, *6*(3), 283-317.

Latour, T., & Martin, R. (2007). TAO, An Open and Versatile Computer-Based Assessment Platform Based on Semantic Web Technology. ECRIM News, *71*, 32-33.

Martin, R., Latour, T., Burton, R., Busana, G., & Vandenabeele, L. (2005). Covering Different Levels of Evaluation Needs by an Internet-Based Computer-Assisted Testing Framework for Collaborative Distributed Test Development and Delivery. Presented at the World Conference on Educational Multimedia, Hypermedia & Telecommunications, Montréal, Canada.

Oldenburg, R. (2005). Bidirektionale Verknüpfung von CAS und DGS – Analysen und Perspektiven. In G. Graumann (Ed.), *Beiträge zum Mathematikunterricht 2005*, Beiträge zum Mathematikunterricht (pp. 424-427). Presented at the 39. Tagung für Didaktik der Mathematik, Bielefeld, Deutschland: Franzbecker. Retrieved from http://www.mathematik.uni-dortmund.de/didaktik/BzMU/BzMU2005/Beitraege/oldenburg-gdm05.pdf

Pellegrino, J. W., Chudowsky, N., & Glaser, R. (Eds.). (2001). *Knowing What Students Know: The Science and Design of Educational Assessment*. The National Academic Press.

Plichart, P., Jadoul, R., Vandenabeele, L., & Latour, T. (2004). TAO, A Collective Distributed Computer-Based Assessment Framework Built on Semantic Web Standards. Presented at the International Conference on Advances in Intelligent Systems – Theory and Application, Luxembourg, Luxembourg. Retrieved from https://www.tao.lu/downloads/publications/AISTA04-paper244-TAO.pdf

Sangwin, C. J. (2007). A brief review of GeoGebra: dynamic mathematics. *MSOR Connections*, *7*(2), 36-38.

Schumann, H. (1991). *Schulgeometrisches Konstruieren mit dem Computer : Beiträge zur Didaktik des interaktiven Konstruierens*. (R. Baumann, K. Keil, L. H. Klingen, K. Menzel, & R. Thode, Eds.)ComputerPraxis im Unterricht. Stuttgart: Metzler und Teubner. Retrieved from http://www.mathe-schumann.de/computer/

Schumann, H., & Green, D. (2000). New protocols for solving geometric calculation problems incorporating dynamic geometry and computer algebra software. *International Journal of Mathematical Education in Science and Technology*, *31*(3), 319-339.

Straesser, R. (2002). Cabri-géomètre: Does Dynamic Geometry Software (DGS) Change Geometry and its Teaching and Learning? *International Journal of Computers for Mathematical Learning*, *6*(3), 319-333. doi:10.1023/A:1013361712895